Math 120 Exam #2 Key (Spring 2016)

1a We have

$$6x^2 - 7x - 10 = 0 \quad\Rightarrow\quad (6x+5)(x-2) = 0 \quad\Rightarrow\quad 6x+5=0 \quad \text{or} \quad x-2=0,$$

so the solution set is $\left\{-\frac{5}{6}, 2\right\}$

1b We have

$$x^2 - 3x = -\tfrac{7}{3} \quad\Rightarrow\quad x^2 - 3x + \tfrac{9}{4} = -\tfrac{7}{3} + \tfrac{9}{4} \quad\Rightarrow\quad \left(x - \tfrac{3}{2}\right)^2 = -\tfrac{1}{12} \quad\Rightarrow\quad x - \tfrac{3}{2} = \pm\tfrac{1}{2\sqrt{3}}i,$$

so $x = \frac{3}{2} \pm \frac{\sqrt{3}}{6}i$. The solution set is $\left\{\frac{3}{2} + \frac{\sqrt{3}}{6}i,\ \frac{3}{2} - \frac{\sqrt{3}}{6}i\right\}$.

2 If n is the smaller even integer, then $n+2$ is the larger one. We have $(n+2)^2 - n^2 = 84$, which gives $4n + 4 = 84$, and thus $n = 20$. The integers are 20 and 22.

3 If w is the width of the metal sheet, then the length of the sheet is $w + 10$. However, the *box* has width $w - 4$ and length $(w+10) - 4 = w + 6$, and the height must be 2. The volume V of the box is computed as $V = 2(w-4)(w+6)$, but we're also given that $V = 832$. This gives us an equation: $2(w-4)(w+6) = 832$. Hence $w^2 + 2w - 440 = 0$, which leads to $(w+22)(w-20) = 0$ and so $w = -22, 20$. Clearly the width of the original sheet can't be -22 cm, which leaves it to be 20 cm. Dimensions of sheet are 20 cm $\times$ 30 cm.

4

	Rate of Work	Time Worked	Fraction of Job Done
Marcobot	$\frac{1}{4}$	t	$\frac{t}{4}$
Teddy	$\frac{1}{7}$	t	$\frac{t}{7}$

Let t be the time it would take to complete the job of filling the sink. We get

$$\frac{t}{4} + \frac{t}{7} = 1 \quad\Rightarrow\quad 7t + 4t = 28 \quad\Rightarrow\quad t = \frac{28}{11} = 2\frac{6}{11} \text{ hours.}$$

5a We have

$$t(t+2)\left(\frac{t}{t+2} + \frac{1}{t} + 3\right) = t(t+2)\cdot\frac{2}{t(t+2)} \quad\Rightarrow\quad t^2 + (t+2) + 3t(t+2) = 2 \quad\Rightarrow\quad 4t^2 + 7t = 0,$$

giving $t(4t+7) = 0$, and finally $t = 0, -\frac{7}{4}$. But 0 is an extraneous solution, so the solution set is $\left\{-\frac{7}{4}\right\}$.

5b Solving,

$$\sqrt{2x} = x - 4 \quad\Rightarrow\quad 2x = (x-4)^2 \quad\Rightarrow\quad x^2 - 10x + 16 = 0 \quad\Rightarrow\quad (x-8)(x-2) = 0,$$

and so $x = 2$ or $x = 8$. But 2 is extraneous (it gives us $2 = -2$ in the original equation), so solution set is $\{8\}$.

5c We have

$$\sqrt{2x+5} = \sqrt{x+2}+1 \;\Rightarrow\; 2x+5 = (\sqrt{x+2}+1)^2 \;\Rightarrow\; 2x+5 = (x+2)+2\sqrt{x+2}+1$$
$$\Rightarrow\; 2\sqrt{x+2} = x+2 \;\Rightarrow\; 4(x+2) = x^2+4x+4$$
$$\Rightarrow\; x^2-4=0 \;\Rightarrow\; x=\pm 2.$$

Solution set is $\{-2, 2\}$.

5d We have

$$3(r^2)^2 + 10r^2 - 25 = 0 \;\Rightarrow\; (r^2+5)(3r^2-5)=0 \;\Rightarrow\; r^2=-5 \quad \text{or} \quad r^2=\frac{5}{3},$$

so $r = \pm i\sqrt{5}$ or $r = \pm\sqrt{5/3}$. Solution set is $\left\{\pm i\sqrt{5}, \pm\sqrt{5/3}\right\}$.

5e Solving

$$|x+1| = |1-3x| \;\Rightarrow\; x+1 = \pm(1-3x) \;\Rightarrow\; x+1 = 1-3x \quad \text{or} \quad x+1 = -(1-3x).$$

The first equation gives $4x=0$ and thus $x=0$; the second equation gives $x+1=3x-1$ and thus $x=1$. Solution set: $\{0,1\}$.

6a $-18 < x-4 < 12 \;\Rightarrow\; -14 < x < 16$, so solution set is $(-14, 16)$.

6b We get $x^2+5x+7<0$, which is not directly factorable. Consider a completing the square procedure:

$$x^2+5x+7 = \left(x^2+5x+\tfrac{25}{4}\right)+7-\tfrac{25}{4} = \left(x+\tfrac{5}{2}\right)^2+\tfrac{3}{4}.$$

Thus $x^2+5x+7<0$ is equivalent to

$$\left(x+\tfrac{5}{2}\right)^2+\tfrac{3}{4}<0,$$

which has no solution. Solution set is therefore $\varnothing$.

6c Get zero on one side:

$$\frac{4}{2-x} \ge \frac{3}{1-x} \;\Rightarrow\; \frac{4}{2-x}-\frac{3}{1-x} \ge 0 \;\Rightarrow\; -\frac{x+2}{(2-x)(1-x)} \ge 0,$$

or equivalently, multiplying both sides by -1 and noting that $(2-x)(1-x) = (x-2)(x-1)$,

$$\frac{x+2}{(x-2)(x-1)} \le 0.$$

Noting that we cannot have $x-2=0$ or $x-1=0$, there are four cases:

Case 1: $x+2 \le 0$ and $x-2<0$ and $x-1<0$.
Case 2: $x+2 \le 0$ and $x-2>0$ and $x-1>0$.
Case 3: $x+2 \ge 0$ and $x-2<0$ and $x-1>0$.
Case 4: $x+2 \ge 0$ and $x-2>0$ and $x-1<0$.

Solving, we have

$$\begin{array}{lllllll} \text{Case 1:} & x \le -2 & \text{and} & x < 2 & \text{and} & x < 1. \\ \text{Case 2:} & x \le -2 & \text{and} & x > 2 & \text{and} & x > 1. \\ \text{Case 3:} & x \ge -2 & \text{and} & x < 2 & \text{and} & x > 1. \\ \text{Case 4:} & x \ge -2 & \text{and} & x > 2 & \text{and} & x < 1. \end{array}$$

Case 1 gives $x \le -2$, and Case 3 gives $1 < x < 2$. The other cases are impossible. Solution set: $(-\infty, -2] \cup (1, 2)$.

6d Solving,

$$|8x - 3| < 4 \;\Rightarrow\; -4 < 8x - 3 < 4 \;\Rightarrow\; -1 < 8x < 7 \;\Rightarrow\; -\tfrac{1}{8} < x < \tfrac{7}{8}.$$

Solution set: $\left(-\frac{1}{8}, \frac{7}{8}\right)$.

6e Since an absolute value is never less than zero, no matter what real number x is we can be sure that $|3 - 2x| \ge -2$ will be satisfied. Solution set: $(-\infty, \infty)$.

7 Let (x, y) be the unknown endpoint. By the midpoint formula:

$$(-7, 6) = \left(\frac{-9 + x}{2}, \frac{9 + y}{2}\right) \;\Rightarrow\; \frac{-9 + x}{2} = -7 \quad \text{and} \quad \frac{9 + y}{2} = 6.$$

Solving these equations gives $x = -5$ and $y = 3$. Thus the segment's other endpoint is at $(-5, 3)$.

8 The center of the circle is at the midpoint of the segment with endpoints $(-1, 2)$ and $(11, 7)$, which has coordinates

$$\left(\frac{-1 + 11}{2}, \frac{2 + 7}{2}\right) = \left(5, \tfrac{9}{2}\right).$$

The radius of the circle is the distance between $\left(5, \frac{9}{2}\right)$ and one of the endpoints of the segment, such as $(-1, 2)$. This distance is

$$\sqrt{(-1 - 5)^2 + \left(2 - \tfrac{9}{2}\right)^2} = \sqrt{36 + \tfrac{25}{4}} = \sqrt{\tfrac{169}{4}} = \tfrac{13}{2}.$$

Thus the center-radius form of the circle is

$$(x - 5)^2 + \left(y - \tfrac{9}{2}\right)^2 = \tfrac{169}{4}.$$