Math 120 Exam #1 Key (Spring 2016)

1 $x^2 + 7y - z^4 = (-4)^2 + 7(4) - (-2)^4 = 16 + 28 - 16 = 28$

2a $(u^3 - 2u^2 + 5) - 2(-7u^3 + 11u^2) = u^3 - 2u^2 + 5 + 14u^3 - 22u^2 = 15u^3 - 24u^2 + 5$

2b $(3v + 2)(4v^2 - 7v + 6) = 12v^3 - 13v^2 + 4v + 12$

2c $(a - 8b)^2 = a^2 - 16ab + 64b^2$

3 Answer is: $4x^2 + 2x + \dfrac{-7x + 1}{2x^2 - x + 2}$.

$$
\begin{array}{r|rrrrr}
 & & & 4x^2 & +2x & \\
\cline{2-6}
2x^2 - x + 2 & 8x^4 & & +6x^2 & -3x & +1 \\
 & -8x^4 & +4x^3 & -8x^2 & & \\
\cline{2-4}
 & & 4x^3 & -2x^2 & -3x & \\
 & & -4x^3 & +2x^2 & -4x & \\
\cline{3-5}
 & & & & -7x &
\end{array}
$$

4a $10ab - 6b + 35a - 21 = 2b(5a - 3) + 7(5a - 3) = (5a - 3)(2b + 7)$

4b $9z^2 + 4z - 2$ is prime

4c $32a^2 + 48ab + 18b^2 = 2(16a^2 + 24ab + 9b^2) = 2(4a + 3b)^2$

4d $36k^2 - 81\ell^4 = 9(4k^2 - 9\ell^4) = 9(2k - 3\ell^2)(2k + 3\ell^2)$

4e $1000x^3 + 343y^3 = (10x)^3 + (7y)^3 = (10x + 7y)(100x^2 - 70xy + 49y^2)$

5a $\dfrac{q^3 + q^2}{7} \cdot \dfrac{49}{q^4 + q^3} = \dfrac{q^2(q + 1)}{7} \cdot \dfrac{7^2}{q^3(q + 1)} = \dfrac{1}{1} \cdot \dfrac{7}{q} = \dfrac{7}{q}$

5b $\dfrac{x^2 + x - 2}{x^2 + 3x - 4} \div \dfrac{x^2 + 3x + 2}{x^2 + 4x + 3} = \dfrac{(x + 2)(x - 1)}{(x + 4)(x - 1)} \cdot \dfrac{(x + 3)(x + 1)}{(x + 2)(x + 1)} = \dfrac{x + 3}{x + 4}$

6 $\dfrac{5}{12x^2y} - \dfrac{7}{6xy^3} = \dfrac{5}{12x^2y} \cdot \dfrac{y^2}{y^2} - \dfrac{7}{6xy^3} \cdot \dfrac{2x}{2x} = \dfrac{5y^2}{12x^2y^3} - \dfrac{14x}{12x^2y^3} = \dfrac{5y^2 - 14x}{12x^2y^3}$

7 We have

$$\frac{1 - \frac{2}{3x}}{9 - \frac{4}{x^2}} = \frac{1 - \frac{2}{3x}}{9 - \frac{4}{x^2}} \cdot \frac{3x^2}{3x^2} = \frac{3x^2 - 2x}{27x^2 - 12} = \frac{x(3x-2)}{3(3x-2)(3x+2)} = \frac{x}{9x+6}$$

8 $\dfrac{(t^{-1/4}u^{2/3})^{12}}{4t^{-3}} = \dfrac{t^{-3}u^8}{4t^{-3}} = \dfrac{u^8}{4}$

9 $q^{-5} - 8q^{-11} = q^{-11}(q^6 - 8)$.

10a $\sqrt{25j^4k^2} = 5|j^2k| = 5j^2k$, since $j, k > 0$.

10b If $x \geq 0$, since $|x| = x$,

$$\sqrt{8x^2z^8} = 2|x|z^4\sqrt{2} = 2xz^4\sqrt{2}.$$

If $x < 0$, since $|x| = -x$,

$$\sqrt{8x^2z^8} = 2|x|z^4\sqrt{2} = -2xz^4\sqrt{2}.$$

10c $\sqrt[3]{\dfrac{9}{16r^4}} = \dfrac{\sqrt[3]{9}}{\sqrt[3]{16r^4}} = \dfrac{\sqrt[3]{9}}{2r\sqrt[3]{2r}} = \dfrac{\sqrt[3]{9}}{2r\sqrt[3]{2r}} \cdot \dfrac{\sqrt[3]{4r^2}}{\sqrt[3]{4r^2}} = \dfrac{\sqrt[3]{36r^2}}{4r^2}$

10d $\sqrt[5]{\sqrt[6]{60}} = \sqrt[30]{60}$

10e $\sqrt[3]{32} - 5\sqrt[3]{4} - 8\sqrt[3]{108} = 2\sqrt[3]{4} - 5\sqrt[3]{4} - 8 \cdot 3\sqrt[3]{4} = -27\sqrt[3]{4}$

11 $4[2x - (2-x) + 5] = -6x - 28 \;\Rightarrow\; 4(3x+3) = -6x - 28 \;\Rightarrow\; 18x = -40 \;\Rightarrow\; x = -\frac{20}{9}$

12 We have

$$ax + b = 4x - 8a \;\Rightarrow\; ax + 8a = 4x - b \;\Rightarrow\; a(x+8) = 4x - b \;\Rightarrow\; a = \frac{4x - b}{x + 8}.$$

13 Let x be the quantity of 60% acid solution to add. We equate the total amount of pure acid present in the two solutions before mixing with the amount of pure acid present in the final mixture:

$$0.60x + 0.45(400) = 0.55(x + 400).$$

Solving yields $x = 800$. That is, 800 mL of 60% solution must be added.

14 Setting up a table may help:

	Rate	Time	Distance
Bike	r	$\frac{3}{4}$	$\frac{3r}{4}$
Car	r+4.5	$\frac{1}{3}$	$\frac{r+4.5}{3}$

Note the time should be entered in hours. The distances must be equal, so we have

$$\frac{3r}{4} = \frac{r + 4.5}{3},$$

which solves to give $r = 3.6$ mi/hr for the bike speed. Thus the distance to work is $3(3.6)/4 = 2.7$ miles.

15a $(1 - 3i)(2 - 6i) = 2 - 6i - 6i + 18i^2 = -16 - 12i$

15b Noting that $-i^2 = -1$, we have

$$\frac{3 - 2i}{1 + i} \cdot \frac{1 - i}{1 - i} = \frac{3 - 3i - 2i + 2i^2}{1 + i - i - i^2} = \frac{1 - 5i}{2} = \frac{1}{2} - \frac{5}{2}i.$$

15c Noting that $i^3 = i^2 \cdot i = -1 \cdot i = -i$, we have

$$i^{277} = i^{4(56)+3} = i^{4(56)} \cdot i^3 = (i^4)^{56} \cdot i^3 = 1^{56} \cdot (-i) = -i.$$