Math 120 Exam #3 Key (Spring 2014)

1 Solve $3x - 4y = 2$ for y to get $y = \frac{3}{4}x - \frac{1}{2}$, which shows the slope for the line given by $3x - 4y = 2$ to be $\frac{3}{4}$. Since the line L through $(-2, -7)$ is parallel to this line, its slope must also be $\frac{3}{4}$. So the equation for L is $y - (-7) = \frac{3}{4}(x - (-2))$, or

$$y = \frac{3}{4}x - \frac{11}{2}.$$

2 Solve $8x - 3y = 6$ for y to get $y = \frac{8}{3}x - 2$, which shows the slope for the line given by $8x - 3y = 6$ to be $\frac{8}{3}$. Since the line L through $(2, -4)$ is perpendicular to this line, its slope must also be $-\frac{3}{8}$. So the equation for L is $y - (-4) = -\frac{3}{8}(x - 2)$, or

$$y = -\frac{3}{8}x - \frac{13}{4}.$$

3 Domain is $[-6, 6]$ and range is $[-8, 8]$. Relation is not a function since it contains two distinct ordered pairs having the same first component value: $(0, 8)$ and $(0, -8)$.

4 $f(-8) = (-8)^2 + \sqrt[3]{-8} = 64 + (-2) = 62$ and $f(c) = c^2 + \sqrt[3]{c}$

5a Domain and range are both $(-\infty, \infty)$

5b Domain is $(-\infty, \infty)$ and range is $[19, \infty)$

6a $\mathrm{Dom}(\alpha) = \left\{x \mid x \neq \frac{2}{3}\right\} = \left(-\infty, \frac{2}{3}\right) \cup \left(\frac{2}{3}, \infty\right)$

6b $\mathrm{Dom}(\beta) = \{x \mid x - 5 \geq 0\} = \{x \mid x \geq 5\} = [5, \infty)$

6c $\mathrm{Dom}(\gamma) = \{x \mid 64 - x^2 \geq 0\} = \{x \mid x^2 \leq 64\} = \{x \mid -8 \leq x \leq 8\} = [-8, 8]$

7a By definition we have

$$(\alpha + \gamma)(x) = \alpha(x) + \gamma(x) = \frac{x+1}{3x-2} + \sqrt{64 - x^2}.$$

Also

$$\mathrm{Dom}(\alpha + \gamma) = \mathrm{Dom}(\alpha) \cap \mathrm{Dom}(\gamma) = \left[\left(-\infty, \tfrac{2}{3}\right) \cup \left(\tfrac{2}{3}, \infty\right)\right] \cap [-8, 8] = \left[-8, \tfrac{2}{3}\right) \cup \left(\tfrac{2}{3}, 8\right].$$

7b We find

$$(\alpha/\beta)(x) = \alpha(x)/\beta(x) = \frac{x+1}{3x-2} \cdot \frac{1}{\sqrt{x-5}} = \frac{x+1}{(3x-2)\sqrt{x-5}},$$

and

$$\begin{aligned}\operatorname{Dom}(\alpha/\beta) &= \{x : x \in \operatorname{Dom}(\alpha) \cap \operatorname{Dom}(\beta) \ \text{ and } \ \beta(x) \neq 0\} \\ &= \left\{x : x \in \left(-\infty, \tfrac{2}{3}\right) \cup \left(\tfrac{2}{3}, \infty\right) \ \text{ and } \ x \in [5,\infty) \ \text{ and } \ x \neq 5\right\} \\ &= (5, \infty).\end{aligned}$$

7c We find

$$(\beta \circ \beta)(x) = \beta(\beta(x)) = \beta\left(\sqrt{x-5}\right) = \sqrt{\sqrt{x-5}-5},$$

and the domain is given by

$$\begin{aligned}\operatorname{Dom}(\beta \circ \beta) &= \{x : x \in \operatorname{Dom}(\beta) \ \text{ and } \ \beta(x) \in \operatorname{Dom}(\beta)\} \\ &= \left\{x : x \geq 5 \ \text{ and } \ \sqrt{x-5} \geq 5\right\} \\ &= \{x : x \geq 5 \ \text{ and } \ x \geq 30\} \\ &= \{x : x \geq 30\} = [30, \infty).\end{aligned}$$

7d We find

$$(\beta \circ \gamma)(x) = \beta(\gamma(x)) = \beta\left(\sqrt{64-x^2}\right) = \sqrt{\sqrt{64-x^2}-5},$$

and the domain is given by

$$\begin{aligned}\operatorname{Dom}(\beta \circ \gamma) &= \{x \mid x \in \operatorname{Dom}(\gamma) \ \text{ and } \ \gamma(x) \in \operatorname{Dom}(\beta)\} \\ &= \left\{x : -8 \leq x \leq 8 \ \text{ and } \ \sqrt{64-x^2} \geq 5\right\} \\ &= \left\{x : -8 \leq x \leq 8 \ \text{ and } \ x^2 \leq 39\right\} \\ &= \left\{x : -8 \leq x \leq 8 \ \text{ and } \ -\sqrt{39} \leq x \leq \sqrt{39}\right\} \\ &= \left[-\sqrt{39}, \sqrt{39}\right].\end{aligned}$$

8 Let $f(x) = 18/x^{10}$ and $g(x) = 7 - 2x$. Then

$$(f \circ g)(x) = f(g(x)) = f(7-2x) = \frac{18}{(7-2x)^{10}} = H(x).$$

9 Suppose that $f(a) = f(b)$. Then

$$2a^3 - 1 = 2b^3 - 1 \quad \Rightarrow \quad 2a^3 = 2b^3 \quad \Rightarrow \quad a^3 = b^3 \quad \Rightarrow \quad a = b.$$

Therefore f is one-to-one.

10 $g(2) = 52 = g(-2)$

11a Suppose that $f(x) = y$. Then

$$y = \frac{x+2}{1-3x} \quad\Rightarrow\quad y - 3xy = x + 2 \quad\Rightarrow\quad 3xy + x = y - 2 \quad\Rightarrow\quad x = \frac{y-2}{3y+1},$$

and since $f^{-1}(y) = x$ by definition, it follows that

$$f^{-1}(y) = \frac{y-2}{3y+1}.$$

11b $\mathrm{Ran}(f) = \mathrm{Dom}(f^{-1}) = (-\infty, -1/3) \cup (-1/3, \infty)$

11c $\mathrm{Ran}(f^{-1}) = \mathrm{Dom}(f) = (-\infty, 1/3) \cup (1/3, \infty)$