MATH 120 EXAM #2 KEY (SPRING 2014)

1a From $9x^2 - 12x + 4 = 0$ we have $(3x-2)(3x-2) = 0$, which leads to $3x - 2 = 0$ and thus $x = 2/3$. Solution set: $\{2/3\}$.

1b We have $x+5 = \pm\sqrt{-3} = \pm i\sqrt{3}$, and so $x = -5 \pm \sqrt{3}i$. Solution set: $\{-5-\sqrt{3}i, -5+\sqrt{3}i\}$.

1c Divide by -3 to get $x^2 - 3x = -7/3$. Completing the square yields

$$x^2 - 3x + (-3/2)^2 = -7/3 + (-3/2)^2,$$

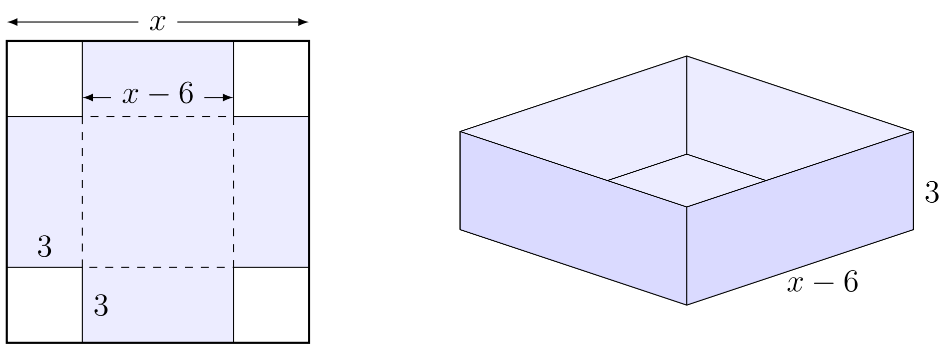
or $(x - 3/2)^2 = -1/12$. From this comes $x - 3/2 = \pm\sqrt{-1/12} = \pm i\sqrt{3}/6$, and so the solution set is

$$\left\{ \frac{3}{2} - \frac{\sqrt{3}}{6}i, \ \frac{3}{2} + \frac{\sqrt{3}}{6}i \right\}.$$

2 Write as $2\pi r^2 + 2\pi hr - S = 0$, so

$$r = \frac{-2\pi h \pm \sqrt{(2\pi h)^2 - 4(2\pi)(-S)}}{2(2\pi)} = \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 + 8\pi S}}{4\pi} = \frac{-\pi h \pm \sqrt{\pi^2 h^2 + 2\pi S}}{2\pi}.$$

3 Let x be the length of the sides of the square base. Once the corners are cut away and the flaps are folded up (see figure below), a box results with a square base of length $x - 6$.



The volume of the box is $3(x-6)(x-6) = 48$, or $(x-6)^2 = 16$. Solving the equation gives $x - 6 = \pm\sqrt{16} = \pm 4$, or $x = 2, 10$. However, we cannot have $x = 2$ since it results in a flap width of -4! So we must have $x = 10$. That is, the cardboard should be 10 cm $\times$ 10 cm.

4 If x is the length of the shorter leg of the right triangle, then $x + 700$ is the length of the longer leg, and the length of the hypotenuse is $(x + 700) + 100 = x + 800$. By the Pythagorean Theorem:

$$x^2 + (x + 700)^2 = (x + 800)^2.$$

Expanding and collecting terms gives $x^2 - 200x - 150,000 = 0$, and thus $(x-500)(x+300) = 0$. Two solutions result: $x = -300$ and $x = 500$. We reject -300 since the length cannot be negative, which leaves $x = 500$ as the only physically relevant solution. The lengths of the triangle's sides are thus 500, 1200, and 1300. The total length of the walkway is the sum of these figures:

$$500 + 1200 + 1300 = 3000 \text{ m}.$$

5 The equation is $t/7 - t/9 = 1$, which becomes $9t - 7t = 63$, and thus $t = 63/2$. That is, it will take $31\frac{1}{2}$ minutes to fill the sink.

6a Multiply by $(2x-1)^2$ to get $2(2x-1)^2 = 3(2x-1) - 1$, which becomes $4x^2 - 7x + 3 = 0$. Factoring gives $(4x-3)(x-1) = 0$, yielding $x = \frac{3}{4}, 1$. Both solutions are valid, so $\{\frac{3}{4}, 1\}$ is the solution set.

6b Multiply by $y - 3$ to get $y = 3 + 3(y-3)$, which yields $y = 3$. But 3 is extraneous, so the solution set is $\varnothing$.

6c Square both sides to obtain

$$3x = (5x+1) - 2\sqrt{5x+1} + 1.$$

Rearranging and dividing by 2 gives $\sqrt{5x+1} = x+1$. Squaring again gives $5x+1 = x^2+2x+1$, then $x^2 - 3x = 0$, and finally $x = 0, 3$. Both solutions are valid, so the solution set is $\{0, 3\}$.

6d Square both sides to get $(3 - \sqrt{x})^2 = 2\sqrt{x} - 3$, or

$$9 - 6\sqrt{x} + x = 2\sqrt{x} - 3.$$

This becomes $8\sqrt{x} = x + 12$, and squaring again yields $64x = x^2 + 24x + 144$. So we have $x^2 - 40x + 144 = 0$, and factoring results in $(x-36)(x-4) = 0$ and finally $x = 4, 36$. But 36 is extraneous, so the solution set is $\{4\}$.

6e Letting $u = (z+2)^2$, the equation becomes quadratic: $6u^2 - 11u + 4 = 0$. Factoring yields $(3u-4)(2u-1) = 0$, or $u = \frac{4}{3}, \frac{1}{2}$. Now,

$$(z+2)^2 = \frac{4}{3} \quad \Rightarrow \quad z + 2 = \pm\frac{2}{\sqrt{3}} \quad \Rightarrow \quad z = -2 \pm \frac{2}{\sqrt{3}},$$

and

$$(z+2)^2 = \frac{1}{2} \quad \Rightarrow \quad z + 2 = \pm\frac{1}{\sqrt{2}} \quad \Rightarrow \quad z = -2 \pm \frac{1}{\sqrt{2}}.$$

Solution set: $\{-2 \pm 2/\sqrt{3}, -2 \pm 1/\sqrt{2}\}$.

6f We have

$$\frac{5}{t-3} = 10 \quad \text{or} \quad \frac{5}{t-3} = -10.$$

Solving these equations gives the solution set $\{\frac{7}{2}, \frac{5}{2}\}$.

7a Simplifying yields $8x - 3 \le 3x - 7$, and then $5x \le -4$, and finally $x \le -4/5$. Solution set: $(-\infty, -4/5]$.

7b Multiply by 20 to get $-10 < 4(4 - 3x) \le 5$, whence

$$-10 < 16 - 12x \le 5 \quad \Rightarrow \quad -26 < -12x \le -11 \quad \Rightarrow \quad 13/6 > x \ge 11/12.$$

Solution set: $\left[\frac{11}{12}, \frac{13}{6}\right)$.

7c Factoring, we get $(x+2)(x+3) > 0$. There are two cases to consider.

Case 1: $x+2 > 0$ & $x+3 > 0$. This gives $x > -2$ & $x > -3$, which is equivalent to $x > -2$.

Case 2: $x+2 < 0$ & $x+3 < 0$. This gives $x < -2$ & $x < -3$, which is equivalent to $x < -3$.

Thus we may have $x < -3$ or $x > -2$. Solution set: $(-\infty, -3) \cup (-2, \infty)$.

7d Manipulate without multiplying by an expression involving x:

$$\frac{2x+1}{x-5} \le 3 \quad \Leftrightarrow \quad \frac{2x+1}{x-5} - 3 \le 0 \quad \Leftrightarrow \quad \frac{2x+1-3(x-5)}{x-5} \le 0 \quad \Leftrightarrow \quad \frac{16-x}{x-5} \le 0.$$

There are two cases to consider.

Case 1: $16 - x \ge 0$ & $x - 5 < 0$. This gives $x \le 16$ & $x < 5$, which is equivalent to $x < 5$. (Note that we cannot have $x = 5$ since division by zero would result.)

Case 2: $16 - x \le 0$ & $x - 5 > 0$. This gives $x \ge 16$ & $x > 5$, which is equivalent to $x \ge 16$. (Letting $x = 16$ results in no division by zero.)

Thus we may have $x < 5$ or $x \ge 16$. Solution set: $(-\infty, 5) \cup [16, \infty)$.

7e We have

$$|4 - 3x| > 2 \quad \Leftrightarrow \quad 4 - 3x > 2 \quad \text{or} \quad 4 - 3x < -2 \quad \Leftrightarrow \quad x < 2/3 \quad \text{or} \quad x > 2.$$

Solution set: $(-\infty, 2/3) \cup (2, \infty)$.

7f We have

$$|5 - x| \le 12 \quad \Leftrightarrow \quad -12 \le 5 - x \le 12 \quad \Leftrightarrow \quad -17 \le -x \le 7 \quad \Leftrightarrow \quad -7 \le x \le 17.$$

Solution set: $[-7, 17]$.

8 We find the distance between the points:

$$\begin{aligned} d(P,Q) &= \sqrt{4^2 + 8^2} = \sqrt{80} = 4\sqrt{5} \\ d(P,R) &= \sqrt{6^2 + 12^2} = \sqrt{180} = 6\sqrt{5} \\ d(Q,R) &= \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5} \end{aligned}$$

Since $d(P,Q) + d(Q,R) = d(P,R)$ (i.e. the two smaller distances equal the largest distance), the points are collinear.

9 Complete the square for each variable as follows:

$$(x^2 - 12x + 36) + (y^2 + 10y + 25) = -25 + 36 + 25.$$

Thus we have $(x-6)^2 + (y+5)^2 = 36$, which is a circle with center at $(6, -5)$ and radius 6.