Math 120 Exam #4 Key (Spring 2013)

1 We have

$$\begin{array}{r|rrr|r} 1 & 3 & 0 & -4 & 2 \\ & & 3 & 3 & -1 \\ \hline & 3 & 3 & -1 & 1 \end{array} \quad \longrightarrow \quad 3x^2+3x-1+\frac{1}{x-1}.$$

2 Divide $f(x)$ by $x-(1+i)$:

$$\begin{array}{r|rrr|r} 1+i & 2 & 3-2i & -8-5i & 3+3i \\ & & 2+2i & 5+5i & -3-3i \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

It follows that

$$f(x) = [x-(1+i)](2x^2+5x-3),$$

and thus

$$f(x) = (2x-1)(x+3)(x-1-i).$$

3a $\dfrac{\text{Factor of } -8}{\text{Factor of } 1} = \pm 1, \pm 2, \pm 4, \pm 8.$

3b The division

$$\begin{array}{r|rrr|r} 1 & 1 & 5 & 2 & -8 \\ & & 1 & 6 & 8 \\ \hline & 1 & 6 & 8 & 0 \end{array}$$

show that 1 is a zero for f, and we obtain the factorization

$$f(x) = (x-1)(x^2+6x+8) = (x-1)(x+2)(x+4).$$

So the zeros of f are: 1, -2, -4.

3c $f(x) = (x-1)(x+2)(x+4)$.

4 We must have $f(x) = c(x+2)(x-1)(x-4)$, with c such that

$$f(2) = c(2+2)(2-1)(2-4) = -8c = 12.$$

Clearly $c = -3/2$ is required, so

$$f(x) = -\frac{3}{2}(x+2)(x-1)(x-4) = -\frac{3}{2}x^3 + \frac{9}{2}x^2 + 9x - 12.$$

5 By the Conjugate Zeros Theorem we must have $2+i$ as a zero also, in order to have rational coefficients. So

$$f(x) = (x+1)[x-(2-i)][x-(2+i)] = x^3-3x^2+x+5.$$

6a We have

$$\text{Dom}(f) = \{x : x^2 - x - 2 \neq 0\} = \{x : (x-2)(x+1) \neq 0\} = \{x : x \neq -1, 2\}.$$

6b The x-intercepts of f are the points $(x, f(x))$ where $f(x) = 0$:

$$\frac{x^2}{(x-2)(x+1)} = 0 \quad\Rightarrow\quad x^2 = 0 \quad\Rightarrow\quad x = 0,$$

so $(0,0)$ is the only x-intercept. Since $(0,0)$ is also a y-intercept of f and a function can never have more than one y-intercept, we have found all intercepts.

6c The vertical asymptotes of f are $x = -1$ and $x = 2$.

6d Since $\deg(x^2) = \deg(x^2 - x - 2) = 2$ and

$$\frac{\text{Lead coefficient of } x^2}{\text{Lead coefficient of } x^2 - x - 2} = \frac{1}{1} = 1,$$

we conclude that $y = 1$ is a horizontal asymptote for f.

6e The graph of f intersects the horizontal asymptote $y = 1$ if there is some $x \in \text{Dom}(f)$ for which $f(x) = 1$. This results in the equation

$$\frac{x^2}{x^2 - x - 2} = 1,$$

giving

$$x^2 = x^2 - x - 2 \quad\Rightarrow\quad x + 2 = 0 \quad\Rightarrow\quad x = -2.$$

Thus the graph of f intersects $y = 1$ at $(-2, 1)$.

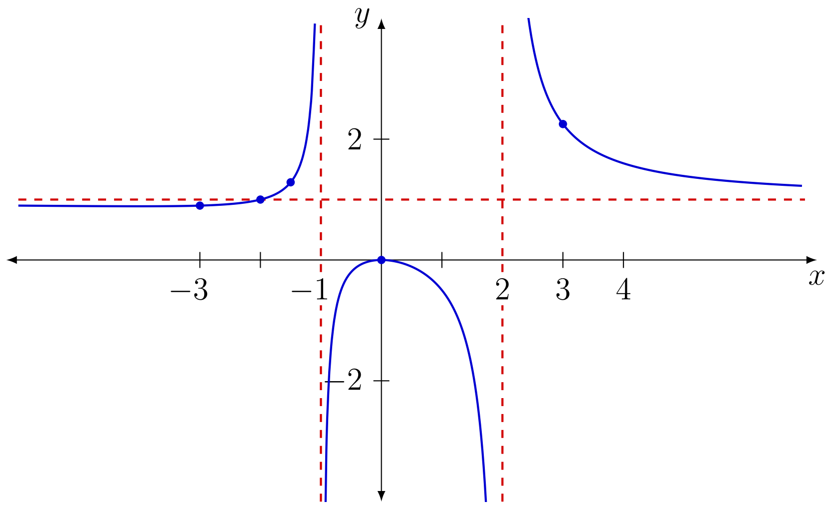
6f The vertical asymptotes partition the plane into three regions:

$$R_1 = \{x : x < -1\}, \quad R_2 = \{x : -1 < x < 2\}, \quad \text{and} \quad R_3 = \{x : x > 2\}.$$

We will want at least one point that lies on the graph of f in each region, and in region R_1 in particular we want points on either side of $(-2, 1)$ where the graph of f intersects the horizontal asymptote $y = 1$. Calculating

$$f(-3) = \tfrac{9}{10}, \quad f\left(-\tfrac{3}{2}\right) = \tfrac{9}{7}, \quad f(3) = \tfrac{9}{4},$$

we obtain the points $\left(-3, \frac{9}{10}\right)$, $\left(-\frac{3}{2}, \frac{9}{7}\right)$, and $\left(3, \frac{9}{4}\right)$. We sketch the graph:



7a Using the formula provided,

$$A(t) = 750\left(1 + \frac{0.08}{4}\right)^{4t} = 750(1.02)^{4t}.$$

7b $750(1.02)^{20} = \$1114.46$ and $750(1.02)^{40} = \$1656.03$.

8a $\log_p(3) = k$

8b $a^{-x} = M$

9 Using a law of logarithms,

$$\ln(x^2 - 9) - \ln(x + 3) = \ln\left(\frac{x^2 - 9}{x + 3}\right) = \ln(x - 3).$$

10a We have

$$5^{4x-7} = 125 \quad \Rightarrow \quad 5^{4x-7} = 5^3 \quad \Rightarrow \quad 4x - 7 = 3 \quad \Rightarrow \quad x = 5/2.$$

10b Take the logarithm of each side:

$$\ln(3^x) = \ln(2^{x-1}) \quad \Rightarrow \quad x\ln 3 = (x-1)\ln 2 \quad \Rightarrow \quad x\ln 3 - x\ln 2 = -\ln 2 \quad \Rightarrow \quad x = \frac{\ln 2}{\ln 2 - \ln 3}.$$

10c Convert to an exponential equation:

$$\log_2(10+3x)=5 \;\;\Rightarrow\;\; 2^5=10+3x \;\;\Rightarrow\;\; 3x=22 \;\;\Rightarrow\;\; x=22/3.$$

10d Consolidate logarithms:

$$\log_2(x+1)+\log_2(x-1)=3 \;\;\Rightarrow\;\; \log_2(x+1)(x-1)=3 \;\;\Rightarrow\;\; 2^3=x^2-1 \;\;\Rightarrow\;\; x=\pm 3.$$

But -3 is an extraneous solution (it results in the logarithm of a negative number in the original equation), so $x=3$ is the only solution.

11 Using the appropriate formula, we have

$$7500=5000\left(1+\frac{0.09}{12}\right)^{12t} \;\;\Rightarrow\;\; 1.0075^{12t}=1.5 \;\;\Rightarrow\;\; \ln(1.0075^{12t})=\ln 1.5,$$

and so

$$12t\ln 1.0075=\ln 1.5 \;\;\Rightarrow\;\; t=\frac{\ln 1.5}{12\ln 1.0075}\approx 4.522.$$

It will take about 4.5 years.

12 Whatever the starting amount P is, we want to find the time t at which $A=2P$. Using the appropriate formula,

$$2P=Pe^{0.036t} \;\;\Rightarrow\;\; e^{0.036t}=2 \;\;\Rightarrow\;\; 0.036t=\ln 2 \;\;\Rightarrow\;\; t=\frac{\ln 2}{0.036}\approx 19.254.$$

It will take about 19.3 years.

13a $n(4)=12e^{0.012(4)}\approx 12.59$, or $12,590,000$.

13b Find t for which $n(t)=35$, which results in the equation $12e^{0.012t}=35$. Solving:

$$e^{0.012t}=35/12 \;\;\Rightarrow\;\; 0.012t=\ln(35/12) \;\;\Rightarrow\;\; t=\frac{\ln(35/12)}{0.012}\approx 89.2.$$

It will take about 89.2 years.