Math 120 Exam #3 Key (Spring 2013)

1 Solve $3x - 4y = 2$ for y to get $y = \frac{3}{4}x - \frac{1}{2}$, which shows the slope for the line given by $3x - 4y = 2$ to be $\frac{3}{4}$. Since the line L through $(-2, -7)$ is parallel to this line, its slope must also be $\frac{3}{4}$. So the equation for L is $y - (-7) = \frac{3}{4}(x - (-2))$, or $y = \frac{3}{4}x - \frac{11}{2}$.

2 Solve $8x - 3y = 6$ for y to get $y = \frac{8}{3}x - 2$, which shows the slope for the line given by $8x - 3y = 6$ to be $\frac{8}{3}$. Since the line L through $(2, -4)$ is perpendicular to this line, its slope must also be $-\frac{3}{8}$. So the equation for L is $y - (-4) = -\frac{3}{8}(x - 2)$, or $y = -\frac{3}{8}x - \frac{13}{4}$.

3 Domain is $[-3, 3]$ and range is $[-4, 4]$. Relation is not a function since it contains two distinct ordered pairs having the same first component value: $(0, 4)$ and $(0, -4)$.

4 By direct substitution,

$$r(7) = 7^2 + \sqrt{7+2} = 52 \quad \text{and} \quad r(-1) = (-1)^2 + \sqrt{-1+2} = 1 + \sqrt{1} = 2.$$

5 We have

$$\text{Dom}(p) = (-\infty, \infty) \quad \text{and} \quad \text{Ran}(p) = [-5, \infty).$$

6a We have

$$\text{Dom}(f) = \{x : x + 7 \neq 0\} = \{x : x \neq -7\} = (-\infty, -7) \cup (-7, \infty)$$

6b We have

$$\text{Dom}(g) = \{x : 4 - 5x \geq 0\} = \{x : 5x \leq 4\} = \{x : x \leq 4/5\} = (-\infty, 4/5]$$

6c We have

$$\text{Dom}(h) = \{x : x - 8 \neq 0 \;\&\; x + 9 \geq 0\} = \{x : x \neq 8 \;\&\; x \geq -9\} = [-9, 8) \cup (8, \infty)$$

7a Finding fg:

$$(fg)(x) = f(x)g(x) = \frac{2x-5}{x+7} \cdot \sqrt{4-5x} = \frac{(2x-5)\sqrt{4-5x}}{x+7}.$$

Now for the domain:

$$\begin{aligned} \text{Dom}(fg) &= \text{Dom}(f) \cap \text{Dom}(g) = \big((-\infty, -7) \cup (-7, \infty)\big) \cap (-\infty, 4/5] \\ &= (-\infty, -7) \cup (-7, 4/5]. \end{aligned}$$

7b Finding h/f:

$$(h/f)(x) = h(x)/f(x) = \frac{\sqrt{x+9}}{x-8} \div \frac{2x-5}{x+7} = \frac{(x+7)\sqrt{x+9}}{(x-8)(2x-5)}.$$

Now for the domain:

$$\begin{aligned}
\mathrm{Dom}(h/f) &= \{x : x \in \mathrm{Dom}(f) \cap \mathrm{Dom}(h) \ \&\ f(x) \neq 0\} \\
&= \Big\{x : x \in \big((-\infty,-7) \cup (-7,\infty)\big) \cap \big([-9,8) \cup (8,\infty)\big) \ \&\ f(x) \neq 0\Big\} \\
&= \{x : x \in [-9,-7) \cup (-7,8) \cup (8,\infty) \ \&\ x \neq 5/2\} \\
&= [-9,-7) \cup (-7,5/2) \cup (5/2,8) \cup (8,\infty).
\end{aligned}$$

7c Finding $f \circ f$:

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{2x-5}{x+7}\right) = \frac{2\left(\dfrac{2x-5}{x+7}\right) - 5}{\left(\dfrac{2x-5}{x+7}\right) + 7} = -\frac{x+45}{9x+44}.$$

Now for the domain:

$$\begin{aligned}
\mathrm{Dom}(f \circ f) &= \{x : x \in \mathrm{Dom}(f) \ \&\ f(x) \in \mathrm{Dom}(f)\} \\
&= \left\{x : x \neq -7 \ \ \&\ \ \frac{2x-5}{x+7} \neq -7\right\},
\end{aligned}$$

where

$$\frac{2x-5}{x+7} \neq -7 \quad \Leftrightarrow \quad 2x-5 \neq -7x-49 \quad \Leftrightarrow \quad 9x \neq -44 \quad \Leftrightarrow \quad x \neq -\tfrac{44}{9},$$

and so

$$\mathrm{Dom}(f \circ f) = \left\{x : x \neq -7 \ \ \&\ \ x \neq -\tfrac{44}{9}\right\} = (-\infty,-7) \cup \left(-7,-\tfrac{44}{9}\right) \cup \left(-\tfrac{44}{9},\infty\right)$$

7d Finding $g \circ g$:

$$(g \circ g)(x) = g(g(x)) = g\big(\sqrt{4-5x}\big) = \sqrt{4-5\sqrt{4-5x}}.$$

Now for the domain:

$$\begin{aligned}
\mathrm{Dom}(g \circ g) &= \{x : x \in \mathrm{Dom}(g) \ \&\ g(x) \in \mathrm{Dom}(g)\} \\
&= \left\{x : x \in (-\infty,4/5] \ \&\ \sqrt{4-5x} \in (-\infty,4/5]\right\},
\end{aligned}$$

where

$$\sqrt{4-5x} \in (-\infty,4/5] \quad \Leftrightarrow \quad \sqrt{4-5x} \leq 4/5 \quad \Leftrightarrow \quad 0 \leq 4-5x \leq 16/25 \quad \Leftrightarrow \quad \tfrac{84}{125} \leq x \leq \tfrac{4}{5},$$

and so

$$\mathrm{Dom}(g \circ g) = \left\{x : x \in \left(-\infty,\tfrac{4}{5}\right] \ \&\ x \in \left[\tfrac{84}{125},\tfrac{4}{5}\right]\right\} = \left[\tfrac{84}{125},\tfrac{4}{5}\right].$$

8 Let $f(x) = 2/x^{10}$ and $g(x) = 7 - 2x$. Then

$$(f \circ g)(x) = f(g(x)) = f(7-2x) = \frac{2}{(7-2x)^{10}} = T(x).$$

9 Suppose that $f(a) = f(b)$. Then

$$2a^3 - 1 = 2b^3 - 1 \quad\Rightarrow\quad 2a^3 = 2b^3 \quad\Rightarrow\quad a^3 = b^3 \quad\Rightarrow\quad a = b.$$

Therefore f is one-to-one.

10 Since $g(10) = 0 = g(3)$, we conclude that g is not one-to-one.

11a Suppose that $f(x) = y$. Then

$$y = \frac{x+1}{2x-3} \quad\Rightarrow\quad 2xy - 3y = x + 1 \quad\Rightarrow\quad 2xy - x = 3y + 1 \quad\Rightarrow\quad x = \frac{3y+1}{2y-1},$$

and since $f^{-1}(y) = x$ by definition, it follows that

$$f^{-1}(y) = \frac{3y+1}{2y-1}.$$

11b By the definition of f^{-1},

$$\operatorname{Ran}(f) = \operatorname{Dom}(f^{-1}) = \left(-\infty, \tfrac{1}{2}\right) \cup \left(\tfrac{1}{2}, \infty\right)$$

and

$$\operatorname{Ran}(f^{-1}) = \operatorname{Dom}(f) = \left(-\infty, \tfrac{3}{2}\right) \cup \left(\tfrac{3}{2}, \infty\right).$$