Math 120 Exam #1 Key (Spring 2013)

1 $-s^2 - 8t + r^2 = -(-2)^2 - 8(3) + (-4)^2 = -4 - 24 + 16 = -12$

2a $(u^3 - 2u^2 + 5) - 2(-7u^3 + 11u^2) = u^3 - 2u^2 + 5 + 14u^3 - 22u^2 = 15u^3 - 24u^2 + 5$

2b $(3v + 2)(4v^2 - 7v + 6) = 12v^3 - 13v^2 + 4v + 12$

2c $(a - 8b)^2 = a^2 - 16ab + 64b^2$

3 Answer is: $2y^2 + y - 1 + \dfrac{6}{5y + 3}$.

$$
\begin{array}{r|l}
5y+3 & 2y^2 + y - 1 \\
\cline{2-2}
 & 10y^3 + 11y^2 - 2y + 3 \\
 & \underline{-\,10y^3 - 6y^2\phantom{{}+3}} \\
 & 5y^2 - 2y \\
 & \underline{-\,5y^2 - 3y\phantom{{}+3}} \\
 & -\,5y + 3 \\
 & \underline{5y + 3} \\
 & 6
\end{array}
$$

4a $10ab - 6b + 35a - 21 = 2b(5a - 3) + 7(5a - 3) = (5a - 3)(2b + 7)$

4b $9z^2 + 4z - 2$ is prime

4c $32a^2 + 48ab + 18b^2 = 2(16a^2 + 24ab + 9b^2) = 2(4a + 3b)^2$

4d $36k^2 - 81\ell^4 = 9(4k^2 - 9\ell^4) = 9(2k - 3\ell^2)(2k + 3\ell^2)$

4e $1000x^3 + 343y^3 = (10x)^3 + (7y)^3 = (10x + 7y)(100x^2 - 70xy + 49y^2)$

5a $\dfrac{q^3 + q^2}{7} \cdot \dfrac{49}{q^4 + q^3} = \dfrac{q^2(q+1)}{7} \cdot \dfrac{7^2}{q^3(q+1)} = \dfrac{1}{1} \cdot \dfrac{7}{q} = \dfrac{7}{q}$

5b $\dfrac{x^2 + x - 2}{x^2 + 3x - 4} \div \dfrac{x^2 + 3x + 2}{x^2 + 4x + 3} = \dfrac{(x+2)(x-1)}{(x+4)(x-1)} \cdot \dfrac{(x+3)(x+1)}{(x+2)(x+1)} = \dfrac{x+3}{x+4}$

6 $\dfrac{5}{12x^2y} - \dfrac{7}{6xy^3} = \dfrac{5}{12x^2y} \cdot \dfrac{y^2}{y^2} - \dfrac{7}{6xy^3} \cdot \dfrac{2x}{2x} = \dfrac{5y^2}{12x^2y^3} - \dfrac{14x}{12x^2y^3} = \dfrac{5y^2 - 14x}{12x^2y^3}$

7 We have

$$\frac{1-\dfrac{2}{3x}}{9-\dfrac{4}{x^2}} = \frac{1-\dfrac{2}{3x}}{9-\dfrac{4}{x^2}} \cdot \frac{3x^2}{3x^2} = \frac{3x^2-2x}{27x^2-12} = \frac{x(3x-2)}{3(3x-2)(3x+2)} = \frac{x}{9x+6}$$

8 $\dfrac{(r^{-1/5}s^{2/3})^{15}}{r^{-2}} = \dfrac{r^{-3}s^{10}}{r^{-2}} = \dfrac{s^{10}}{r}$

9 $t^{-5} - 3t^{-3} = t^{-5}(1-3t^2)$.

10a $\sqrt{25j^4k^2} = 5j^2|k|$.

10b $\sqrt{8x^5z^8} = 2x^2z^4\sqrt{2x}$

10c $\sqrt[3]{\dfrac{9}{16r^4}} = \dfrac{\sqrt[3]{9}}{\sqrt[3]{16r^4}} = \dfrac{\sqrt[3]{9}}{2r\sqrt[3]{2r}} = \dfrac{\sqrt[3]{9}}{2r\sqrt[3]{2r}} \cdot \dfrac{\sqrt[3]{4r^2}}{\sqrt[3]{4r^2}} = \dfrac{\sqrt[3]{36r^2}}{4r^2}$

10d $\sqrt[4]{\sqrt[3]{5}} = \sqrt[12]{5}$

10e $\sqrt[3]{32} - 5\sqrt[3]{4} + 2\sqrt[3]{108} = 2\sqrt[3]{4} - 5\sqrt[3]{4} + 2 \cdot 3\sqrt[3]{4} = 3\sqrt[3]{4}$

11 $4[2x-(3-x)+5] = -6x-28 \;\Rightarrow\; 4(3x+2) = -6x-28 \;\Rightarrow\; 18x = -36 \;\Rightarrow\; x = -2$

12 $ax+b = 3(x-a) \;\Rightarrow\; ax+b = 3x-3a \;\Rightarrow\; ax+3a = 3x-b \;\Rightarrow\; a(x+3) = 3x-b \;\Rightarrow\; a = \dfrac{3x-b}{x+3}$

13 Let x be the quantity of 60% acid solution to add. We equate the total amount of pure acid present in the two solutions before mixing with the amount of pure acid present in the final mixture:

$$0.60x + 0.45(400) = 0.55(x+400).$$

Solving yields $x = 800$. That is, 800 mL of 60% solution must be added.

14 Let x be the amount of money invested at 4.5%, so that $12,000 - x$ is the amount invested at 4%. Add the interest from each investment to get the total interest of \$525:

$$0.045x + 0.04(12,000 - x) = 525.$$

Solving yields $x = 9,000$. That is, \$9,000 is invested at 4.5% and \$3,000 at 4%.

15a $(3-6i)-(-8-13i)=11+7i$

15b $(4-3i)(2+9i)=8+36i-6i-27i^2=35+30i$

15c $\dfrac{3-2i}{2-i}\cdot\dfrac{2+i}{2+i}=\dfrac{6+3i-4i-2i^2}{4+2i-2i-i^2}=\dfrac{8-i}{5}=\dfrac{8}{5}-\dfrac{1}{5}i$

15d $i^{377}=i^{4(94)+1}=i^{4(94)}\cdot i^1=(i^4)^{94}\cdot i=(1)^{94}\cdot i=1\cdot i=i$