Math 120 Exam #2 Key (Spring 2012)

1a. $12x^2 - 8x + 1 = 0 \Rightarrow (6x-1)(2x-1) = 0 \Rightarrow 6x - 1 = 0$ or $2x - 1 = 0 \Rightarrow x = 1/6$ or $x = 1/2$, so solution set is $\{1/6, 1/2\}$.

1b. $x^2 - 2x = \frac{3}{2} \Rightarrow x^2 - 2x + 1 = \frac{3}{2} + 1 \Rightarrow (x-1)^2 = \frac{5}{2} \Rightarrow x - 1 = \pm\sqrt{\frac{5}{2}} \Rightarrow x = 1 \pm \frac{\sqrt{10}}{2}$. Solution set is $\left\{1 \pm \frac{\sqrt{10}}{2}\right\}$.

1c. $x^3 + 4^3 = 0 \Rightarrow (x+4)(x^2 - 4x + 16) = 0 \Rightarrow x + 4 = 0$ or $x^2 - 4x + 16 = 0$, and so by the quadratic formula we obtain $x = -4$ or $x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(16)}}{2(1)} = \dfrac{4 \pm \sqrt{-48}}{2} = \dfrac{4 \pm 4i\sqrt{3}}{2} = 2 \pm 2i\sqrt{3}$. Solution set is $\{-4, 2 \pm 2i\sqrt{3}\}$.

2. Volume=(depth)(length)(width), so if ℓ is the length we have $2.3125\ell(\ell - 3.1875) = 182.742$, which is the quadratic equation $2.3125\ell^2 - 7.3711\ell - 182.742 = 0$. By the quadratic formula we get

$$\ell = \frac{7.3711 \pm \sqrt{(-7.3711)^2 - 4(2.3125)(-182.742)}}{2(2.3125)} = \frac{7.3711 \pm 41.7696}{4.6250} = 10.625, -7.438$$

Length cannot be negative, so we conclude that $\ell = 10.625$ inches. Width is thus $10.625 - 3.1875 = 7.438$ inches. Dimensions of the box are $10.625 \times 7.438 \times 2.3125$ inches.

3. Let x be the width of the border. Since the area of the kitchen is 120 ft^2 and the area of the border must be 21 ft^2, the area of the vinyl inside the border must be 99 ft^2. The dimensions of the rectangle inside the border are $12 - 2x$ by $10 - 2x$, and so $(12-2x)(10-2x) = 99$ is the equation. A little manipulation yields $4x^2 - 44x + 21 = 0$, which factors as $(2x-1)(2x-21) = 0$, and so either $x = 1/2$ or $x = 21/2$. But a border width of 21/2 ft is impossible, so we conclude that the border must be 1/2 ft wide.

4a. Multiply by $(x+2)(x+4)$ to get $2(x+4) + (x+2) = 4$, so $3x = -6$ and we get $x = -2$. But this solution is extraneous, so solution set is $\varnothing$.

4b. Square both sides to get $2x + 3 = (x+2)^2$, so $2x + 3 = x^2 + 4x + 4 \Rightarrow x^2 + 2x + 1 = 0 \Rightarrow (x+1)^2 = 0 \Rightarrow x = -1$. Solution set is $\{-1\}$.

4c. Square both sides to get $(3 - \sqrt{x})^2 = 2\sqrt{x} - 3$. Then $9 - 6\sqrt{x} + x = 2\sqrt{x} - 3 \Rightarrow 8\sqrt{x} = x + 12$, and squaring again gives $64x = (x+12)^2 \Rightarrow 64x = x^2 + 24x + 144 \Rightarrow x^2 - 40x + 144 = 0 \Rightarrow (x-36)(x-4) = 0 \Rightarrow x = 4, 36$. But 36 is extraneous, so solution set is $\{4\}$.

4d. Letting $u = (x+1)^{1/5}$, the equation becomes $u^2 - 3u + 2 = 0$, which factors as $(u-2)(u-1) = 0$ so that $u = 1, 2$. From $u = 1$ comes $(x+1)^{1/5} = 1$, yielding $x + 1 = 1$ and finally $x = 0$. From $u = 2$ comes $(x+1)^{1/5} = 2$, yielding $x + 1 = 32$ and finally $x = 31$. Solution set is $\{0, 31\}$.

4e. Either $2x - 3 = 5x + 4$ or $2x - 3 = -(5x+4)$. Solving the first equation yields $x = -7/3$, and solving the second equation yields $x = -1/7$. Solution set is $\{-7/3, -1/7\}$.

5a. $6x - 2x - 3 \geq 3x - 5 \;\Rightarrow\; 4x - 3 \geq 3x - 5 \;\Rightarrow\; x \geq -2$, so solution set is $[-2, \infty)$.

5b. $-9 < x - 1 < 6 \;\Rightarrow\; -8 < x < 7$, so solution set is $(-8, 7)$.

5c. $6x^2 - 11x - 10 < 0 \;\Rightarrow\; (3x+2)(2x-5) < 0$. Case 1: $3x + 2 < 0$ & $2x - 5 > 0$, which leads to a contradiction. Case 2: $3x + 2 > 0$ & $2x - 5 < 0$, which leads to $-\frac{2}{3} < x < \frac{5}{2}$. Solution set: $\left(-\frac{2}{3}, \frac{5}{2}\right)$.

5d. $2x^3 - 3x^2 - 5x \leq 0 \;\Rightarrow\; x(2x-5)(x+1) \leq 0$. Case 1: $x \leq 0$, $2x - 5 \geq 0$, $x + 1 \geq 0$, which leads to contradiction. Case 2: $x \geq 0$, $2x - 5 \leq 0$, $x + 1 \geq 0$, which leads to $0 \leq x \leq \frac{5}{2}$. Case 3: $x \geq 0$, $2x - 5 \geq 0$, $x + 1 \leq 0$, again contradictory. Case 4: $x \leq 0$, $2x - 5 \leq 0$, $x + 1 \leq 0$, which leads to $x \leq -1$. Solution set: $(-\infty, -1] \cup \left[0, \frac{5}{2}\right]$.

5e. $\dfrac{10}{2x-3} \leq 5 \;\Rightarrow\; \dfrac{10}{2x-3} - \dfrac{5(2x-3)}{2x-3} \leq 0 \;\Rightarrow\; \dfrac{25-10x}{2x-3} \leq 0$. Case 1: $25 - 10x \leq 0$ & $2x - 3 > 0$, which yields $x \geq \frac{5}{2}$ & $x > \frac{3}{2}$, and therefore $x \geq \frac{5}{2}$. Case 2: $25 - 10x \geq 0$ & $2x - 3 < 0$, which yields $x \leq \frac{5}{2}$ & $x < \frac{3}{2}$, and therefore $x \geq \frac{3}{2}$. Solution set: $\left(-\infty, \frac{3}{2}\right) \cup \left[\frac{5}{2}, \infty\right)$.

5f. $8x - 3 > 5$ or $8x - 3 < -5 \;\Rightarrow\; 8x > 8$ or $8x < -2 \;\Rightarrow\; x > 1$ or $x < -1/4$. So the solution set is $(-\infty, -1/4) \cup (1, \infty)$.

6. Distance $= \sqrt{(6-4)^2 + (-2-6)^2} = \sqrt{2^2 + 8^2} = \sqrt{68} = 2\sqrt{17}$.

7. Three ordered pairs in the solution set are $(0, \sqrt{2})$, $(2, 2)$, $(-1, 1)$. There are many others.

8. $(x^2 - 12x + 36) + (y^2 + 10y + 25) = -25 + 36 + 25 \;\Rightarrow\; (x-6)^2 + (y+5)^2 = 36$, which is a circle with center at $(6, -5)$ and radius 6.