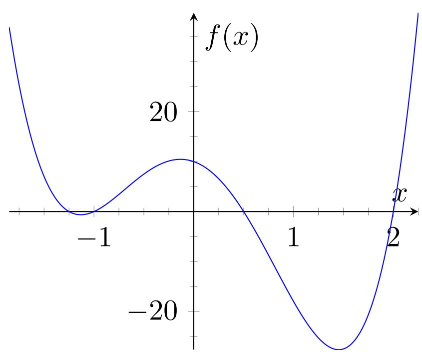
# Math 120 Exam #4 Key (Spring 2011)

**1.** $\begin{array}{r|rrrrr|r} -2 & 1 & 3 & 2 & 2 & 3 & 1 \\ & & -2 & -2 & 0 & -4 & 2 \\ \hline & 1 & 1 & 0 & 2 & -1 & 3 \end{array}$ $\longrightarrow$ so the result is: $x^4 + x^3 + 2x - 1 + \dfrac{3}{x+2}$.

**2a.** $\dfrac{\pm 1, \pm 2, \pm 5, \pm 10}{1, 2, 4, 8} = \pm 1,\ \pm\frac{1}{2},\ \pm\frac{1}{4},\ \pm\frac{1}{8},\ \pm 2, \pm 5,\ \pm\frac{5}{2},\ \pm\frac{5}{4},\ \pm\frac{5}{8},\ \pm 10.$

**2b.** The divisions $\begin{array}{r|rrrr|r} 2 & 8 & -2 & -27 & -7 & 10 \\ & & 16 & 28 & 2 & -10 \\ \hline & 8 & 14 & 1 & -5 & 0 \end{array}$ and $\begin{array}{r|rrr|r} -1 & 8 & 14 & 1 & -5 \\ & & -8 & -6 & 5 \\ \hline & 8 & 6 & -5 & 0 \end{array}$ show that 2 and $-1$ are zeros for the function $f$, and we obtain the factorization $f(x) = (x-2)(x+1)(8x^2 + 6x - 5)$ from the bottom row of numbers in the second synthetic division. Now $f(x) = 0$ implies (by the Zero Factor Principle from way back) that $x - 2 = 0$ or $x + 1 = 0$ or $8x^2 + 6x - 5 = 0$. The last equation yields two new zeros: $\frac{1}{2}$ and $-\frac{5}{4}$. So the zeros of $f$ are: $2,\ -1,\ \frac{1}{2},\ -\frac{5}{4}$.

**2c.** From part (b) it is clear that $f(x) = (x-2)(x+1)(4x+5)(2x-1)$. For giggles the graph of $f$ is included here.




**3.** We must have $f(x) = c(x+2)(x-1)(x-4)$ by the Factor Theorem, where $c$ will be an appropriate constant that will give $f(2) = 16$. But $f(2) = 16$ gives us $c(2+2)(2-1)(2-4) = 16$, or $c = -2$. Hence

$$f(x) = -2(x+2)(x-1)(x-4).$$

**4.** To have real coefficients the Conjugate Zeros Theorem implies that $2 - i$ must also be a zero. Then, by the Factor Theorem, we obtain $f(x) = (x-3)[x-(2+i)][x-(2-i)] = (x-3)(x^2-4x+5)$, or $f(x) = x^3 - 7x^2 + 17x - 15$.

**5a.** Equate bases: $(4^3)^{2x-1} = 4^{3x} \ \Rightarrow\ 4^{6x-3} = 4^{3x} \ \Rightarrow\ 6x - 3 = 3x \ \Rightarrow\ x = 1$.

**5b.** Convert to exponential equation: $8^y = \sqrt[4]{8} \ \Rightarrow\ 8^y = 8^{1/4} \ \Rightarrow\ y = 1/4$.

**5c.** Convert to exponential equation: $x^{-2} = 3 \ \Rightarrow\ x^2 = 1/3 \ \Rightarrow\ x = \pm 1/\sqrt{3}$; but the base of a logarithm cannot be negative, so the only solution is $x = 1/\sqrt{3}$.

**6a.** $6^{x+3} = 5^x \ \Rightarrow\ \ln(6^{x+3}) = \ln(5^x) \ \Rightarrow\ (x+3)\ln 6 = x \ln 5 \ \Rightarrow\ x\ln 5 - x \ln 6 = 3 \ln 6 \ \Rightarrow\ x = \dfrac{3 \ln 6}{\ln 5 - \ln 6} \approx -29.4824$.

**6b.** Since any logarithmic function is one-to-one, we obtain $3x + 8 = 18$ and hence $x = 10/3$.

**6c.** $\log_2 x + \log_2(x+2) = 3 \Rightarrow \log_2[x(x+2)] = 3 \Rightarrow 2^3 = x(x+2) \Rightarrow x^2+2x-8 = 0 \Rightarrow (x+4)(x-2) = 0 \Rightarrow x = -4, 2$. But $-4$ is extraneous since it results in the logarithm of a negative number in the original equation, and so the only solution is $x = 2$.

**7.** $A = P(1 + r/m)^{mt} \Rightarrow 3000 = 1200(1 + r/12)^{12(5)} \Rightarrow (1 + r/12)^{60} = 2.5 \Rightarrow \sqrt[60]{(1 + r/12)^{60}} = \sqrt[60]{2.5} \Rightarrow 1 + r/12 = 1.01539 \Rightarrow r = 0.18466$, and so the interest rate would need to be approximately 18.47%. Good luck with that.

**8a.** Three weeks is 21 days, so the amount is $A(21) = 600e^{-0.0057(21)} = 532.312 \approx 532.3$ grams.

**8b.** The starting amount is $A(0) = 600$ grams, so we find the time $t$ for which $A(t) = 300$: $300 = 600e^{-0.0057t} \Rightarrow e^{-0.0057t} = 0.5 \Rightarrow -0.0057t = \ln 0.5 \Rightarrow t = 121.6$ days.

**9.** Even agents of international mayhem need to know a little math. Our initial amount of kaboomium is $A_0 = 260$ grams, meaning our basic model is $A(t) = 260e^{-kt}$ and it remains to find $k$. We're given that $A(5) = 192$ g, which is to say that $192 = 260e^{-k\cdot 5}$. Solve this equation: $e^{-5k} = 0.7385 \Rightarrow -5k = \ln 0.7385 \Rightarrow k = \frac{\ln 0.7385}{-5} \approx 0.0606$. Thus the model here is $A(t) = 260e^{-0.0606t}$, and it's now possible to find the time $t$ for which just 10 g of kaboomium remains: $10 = 260e^{-0.0606t} \Rightarrow e^{-0.0606t} = 0.0385 \Rightarrow -0.0606t = \ln 0.0385 \Rightarrow t = \frac{\ln 0.0385}{-0.0606} \approx 53.7$ hours. We assume Cobra Commander knows his business. And knowing is half the battle!

**10a.** From the first equation we obtain $x = \frac{1}{2}(3y - 7)$, which can then be substituted into the second equation to give $5 \cdot \frac{1}{2}(3y - 7) + 4y = 17$. Solving this yields $y = 3$. Now put this result into $x = \frac{1}{2}(3y - 7)$ to get $x = 1$. The solution is $(1, 3)$

**10b.** The 3rd equation gives $y = 2x - 4z + 14$. Putting this into the 1st and 2nd equations yields

$$4x - (2x - 4z + 14) + 3z = -2$$

and

$$3x + 5(2x - 4z + 14) - z = 15,$$

respectively. Simplifying gives us the system

$$\begin{cases} 2x + 7z = 12 \\ 13x - 21z = -55 \end{cases}$$

This system solves to give $x = -1$ and $z = 2$. Putting these values into $y = 2x - 4z + 14$ gives $y = 4$. Hence the solution is $(-1, 4, 2)$.