Math 120 Exam #2 Key (Spring 2011)

1a. $5x^2 - 3x - 2 = 0 \;\Rightarrow\; (5x+2)(x-1) = 0 \;\Rightarrow\; 5x+2 = 0$ or $x - 1 = 0$, so $x = 1, -2/5$

1b. $x^2 - 3x = 6 \;\Rightarrow\; x^2 - 3x + 9/4 = 6 + 9/4 \;\Rightarrow\; \left(x - \frac{3}{2}\right)^2 = \frac{33}{4} \;\Rightarrow\; x - \frac{3}{2} = \pm\frac{\sqrt{33}}{2} \;\Rightarrow\; x = \frac{3}{2} \pm \frac{\sqrt{33}}{2}$

2. If w is the width of the metal sheet, then the length of the sheet is $w + 10$. However, the *box* has width $w - 4$ and length $(w+10) - 4 = w + 6$, and the height must be 2. The volume V of the box is computed as $V = 2(w-4)(w+6)$, but we're also given that $V = 832$. This gives us an equation: $2(w-4)(w+6) = 832$. Hence $w^2 + 2w - 440 = 0$, which leads to $(w+22)(w-20) = 0$ and so $w = -22, 20$. Clearly the width of the original sheet can't be -22 cm, which leaves it to be 20 cm. Dimensions of sheet: 20 cm by 30 cm.

3.

	rate	time	distance
Lennier	2.7	t	$2.7t$
Vir	3.3	t	$3.3t$

The distance between Lennier and Vir is the hypotenuse of a right triangle, and we need to find the time t when the distance is 6 miles. By the Pythagorean Theorem we must have $(2.7t)^2 + (3.3t)^2 = 6^2$, giving $18.18t^2 = 36$ and thus $t \approx 1.407$ hours. To the nearest minute: 1 hour, 24 minutes.

4a. $x(x+2)\left(\frac{x}{x+2} + \frac{1}{x} + 3\right) = x(x+2) \cdot \frac{2}{x(x+2)} \;\Rightarrow\; x^2 + (x+2) + 3x(x+2) = 2 \;\Rightarrow\; 4x^2 + 7x = 0 \;\Rightarrow$ $x(4x+7) = 0 \;\Rightarrow\; x = 0, -\frac{7}{4}$. However, the "solution" 0 is extraneous, so the solution set is $\left\{-\frac{7}{4}\right\}$.

4b. $\sqrt{2x} = x - 4 \;\Rightarrow\; 2x = (x-4)^2 \;\Rightarrow\; x^2 - 10x + 16 = 0 \;\Rightarrow\; (x-8)(x-2) = 0 \;\Rightarrow\; x = 2, 8$. But 2 is extraneous (it gives us $2 = -2$ in the original equation), so solution set is $\{8\}$.

4c. $\sqrt{x} = \sqrt{x+3} - 1 \;\Rightarrow\; x = (\sqrt{x+3} - 1)^2 \;\Rightarrow\; x = (x+3) - 2\sqrt{x+3} + 1 \;\Rightarrow\; 2\sqrt{x+3} = 4 \;\Rightarrow$ $4(x+3) = 16 \;\Rightarrow\; x = 1$. Solution set: $\{1\}$.

4d. Let $u = x^2$, so equation becomes $4u^2 + 3u - 1 = 0$, which becomes $(4u-1)(u+1) = 0$ and gives $u = -1, \frac{1}{4}$. Now, $x^2 = -1$ yields $x = \pm i$, and $x^2 = \frac{1}{4}$ yields $x = \pm\frac{1}{2}$. Solution set: $\left\{\pm i, \pm\frac{1}{2}\right\}$.

4e. $|20 - 3x| = 29$ implies that $20 - 3x = \pm 29$; then, $20 - 3x = 29 \;\Rightarrow\; x = -3$, and $20 - 3x = -29 \;\Rightarrow$ $-3x = -49 \;\Rightarrow\; x = \frac{49}{3}$. Solution set: $\left\{-3, \frac{49}{3}\right\}$.

5a. $4x - 3 \geq 3x - 5 \;\Rightarrow\; x \geq -2$, so solution set is $[-2, \infty)$

5b. $-18 < x - 4 < 12 \;\Rightarrow\; -14 < x < 16$, so solution set is $(-14, 16)$

5c. $6x^2 - 11x - 10 < 0 \;\Rightarrow\; (3x+2)(2x-5) < 0$. Case 1: $3x + 2 < 0$ & $2x - 5 > 0$, which leads to a contradiction. Case 2: $3x + 2 > 0$ & $2x - 5 < 0$, which leads to $-\frac{2}{3} < x < \frac{5}{2}$. Solution set: $\left(-\frac{2}{3}, \frac{5}{2}\right)$.

5d. $2x^3 - 3x^2 - 5x \leq 0 \;\Rightarrow\; x(2x-5)(x+1) \leq 0$. Case 1: $x \leq 0$, $2x - 5 \geq 0$, $x + 1 \geq 0$, which leads to contradiction. Case 2: $x \geq 0$, $2x-5 \leq 0$, $x+1 \geq 0$, which leads to $0 \leq x \leq \frac{5}{2}$. Case 3: $x \geq 0$, $2x-5 \geq 0$, $x+1 \leq 0$, again contradictory. Case 4: $x \leq 0$, $2x-5 \leq 0$, $x+1 \leq 0$, which leads to $x \leq -1$. Solution set: $(-\infty, -1] \cup \left[0, \frac{5}{2}\right]$.

5e. $\dfrac{10}{2x-3} \leq 5 \quad \Rightarrow \quad \dfrac{10}{2x-3} - \dfrac{5(2x-3)}{2x-3} \leq 0 \quad \Rightarrow \quad \dfrac{25-10x}{2x-3} \leq 0$. Case 1: $25-10x \leq 0$ & $2x-3 > 0$, which yields $x \geq \frac{5}{2}$ & $x > \frac{3}{2}$, and therefore $x \geq \frac{5}{2}$. Case 2: $25-10x \geq 0$ & $2x-3 < 0$, which yields $x \leq \frac{5}{2}$ & $x < \frac{3}{2}$, and therefore $x \geq \frac{3}{2}$. Solution set: $\left(-\infty, \frac{3}{2}\right) \cup \left[\frac{5}{2}, \infty\right)$.

5f. $|8x-3| < 4 \quad \Rightarrow \quad -4 < 8x-3 < 4 \quad \Rightarrow \quad -1 < 8x < 7 \quad \Rightarrow \quad -\frac{1}{8} < x < \frac{7}{8}$. Solution set: $\left(-\frac{1}{8}, \frac{7}{8}\right)$.

6. $\sqrt{(-4-6)^2 + (3-2)^2} = \sqrt{101}$

7 $(x^2+8x+16) + (y^2-6y+9) = -16+16+9 \quad \Rightarrow \quad (x+4)^2 + (y-3)^2 = 3^2$, which is a circle with center at $(-4, 3)$ and radius 3.

8. $m = \dfrac{8-2}{-3-(-7)} = \dfrac{6}{4} = \dfrac{3}{2}$

9. Using the point-slope formula we get $y-4 = -3(x-2)$, which in standard form becomes $3x+y = 10$.

10. The equation $3x+4y = 1$ becomes $y = -\frac{3}{4}x + \frac{1}{4}$, which indicates a slope of $-\frac{3}{4}$ for the line $3x+4y = 1$. Thus the line perpendicular to it must have slope $\frac{4}{3}$, which, together with the point $(2, 7)$, gives us the equation $y-7 = \frac{4}{3}(x-2)$ and finally $y = \frac{4}{3}x + \frac{13}{3}$.