MATH 120 EXAM #4 KEY (SPRING '09)

1 $f(x) = 4(x^2 - x + \frac{1}{4}) + 3 - 1 = 4(x - \frac{1}{2})^2 + 2$

Vertex: $(\frac{1}{2}, 2)$, Axis: $x = \frac{1}{2}$, Domain: $(-\infty, \infty)$, Range: $[2, \infty)$

2 $\pm i$ are zeros, so x^2+1 is factor. Now...

$\frac{f(x)}{x^2+1} = x^2+10x+26 \Rightarrow f(x) = (x^2+1)(x^2+10x+26)$

From $x^2+10x+26=0$ we get zeros $-5 \pm i$.

Zeros: $\{\pm i, -5 \pm i\}$

3 Possible Rational Zeros: $\frac{\pm 1, \pm 5}{\pm 1, \pm 2} = \pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}$

$$\begin{array}{r|rrrr} -1 & 2 & -9 & -6 & 5 \\ & & -2 & 11 & -5 \\ \hline & 2 & -11 & 5 & 0 \end{array} \checkmark$$

$\rightarrow$ -1 is a zero, and $f(x) = (x+1)(2x^2 - 11x + 5)$

Now, $f(x) = (x+1)(2x-1)(x-5)$, and it's seen that zeros are $-1, \frac{1}{2}, 5$.

4 $f(x) = k(x+2)(x-1)(x-4)$, where $f(2) = 16$ implies: $16 = k(2+2)(2-1)(2-4)$, or $-8k = 16$, or $k = -2$. Thus $f(x) = -2(x+2)(x-1)(x-4) = -2x^3 + 6x^2 + 12x - 16$

5 Zeros must be $2-i$, 5, & $\underline{2+i}$ (Conjugate Zeros Theorem). Then: $f(x) = (x-5)[x-(2-i)][x-(2+i)]$ by Factor Theorem, or $f(x) = (x-5)(x^2-4x+5)$. Hence: $f(x) = x^3 - 9x^2 + 25x - 25$.

6 $A = 32{,}000\left(1 + \frac{0.066}{4}\right)^{4(12)} = \$70{,}194.71$

7 $\log_5\left(\frac{a^{10}}{c^{35}}\right)$

8a $2^{6-3x} = 2^{3(x+1)} \Rightarrow 6 - 3x = 3(x+1) \Rightarrow 6x = 3 \Rightarrow x = \frac{1}{2}$

8b $5^{-x/3} = 5^{-(x+2)} \Rightarrow -\frac{x}{3} = -(x+2) \Rightarrow -2x = 6 \Rightarrow x = -3$

8c $6^x = \frac{1}{216} \Rightarrow 6^x = 6^{-3} \Rightarrow x = -3$

8d $x^{2/3} = 16^{1/3} \Rightarrow x^{2/3} = 4^{2/3} \Rightarrow x = 4$

8e $(0.7)^x = 0.5 \Rightarrow \ln 0.7^x = \ln 0.5 \Rightarrow x \ln 0.7 = \ln 0.5 \Rightarrow x = \ln 0.5 / \ln 0.7 \Rightarrow x \approx 1.943$

8f $\ln 8^{2x+1} = \ln 10^{1-x} \Rightarrow$
$(2x+1)\ln 8 = (1-x)\ln 10 \Rightarrow$
$2x \ln 8 + x \ln 10 = \ln 10 - \ln 8 \Rightarrow$
$x = \frac{\ln(5/4)}{2\ln 8 + \ln 10} = \frac{\ln(1.25)}{\ln(640)} \approx 0.035$

8g $\ln(3x^2 - 13x) = 1 \Rightarrow e^1 = 3x^2 - 13x \Rightarrow$
$3x^2 - 13x - e = 0 \Rightarrow$ (using Quadratic Formula)
$x = \frac{13 \pm \sqrt{169 - 4(3)(-e)}}{2(3)} \approx \frac{13 \pm 14.19927}{6}$
$x \approx \cancel{-0.200}, \boxed{4.533}$

8h $\log\left(\frac{11x+9}{x+3}\right) = 3 \Rightarrow 10^3 = \frac{11x+9}{x+3} \Rightarrow$
$1000x + 3000 = 11x + 9 \Rightarrow 989x = -2991 \Rightarrow$
$x = \cancel{-\frac{2991}{989}} \rightarrow$ extraneous

No solution

9 $0.067A_0 = A_0 e^{-0.0001216t} \Rightarrow$
$e^{-0.0001216t} = 0.067 \Rightarrow -0.0001216t = \ln 0.067$
$\Rightarrow t = \frac{\ln 0.067}{-0.0001216} \approx 22{,}229$ years

10 $2P = Pe^{0.037t} \Rightarrow e^{0.037t} = 2 \Rightarrow$
$0.037t = \ln 2 \Rightarrow t = \ln 2 / 0.037 = 18.73$ yrs.

11 $A(t) = A_0 e^{-kt}$ is the basic model, so with $A_0 = 30$ g we have:
$A(12) = 30e^{-12k} \Rightarrow 28.7 = 30e^{-12k} \Rightarrow$
$e^{-12k} = 0.95667 \Rightarrow -12k = \ln 0.95667 \Rightarrow$
$k = (\ln 0.95667)/-12 = 0.00369$. Thus:
$A(t) = 30e^{-0.00369t}$ is our model.
A century means $t = \mathbf{100}$, so...
$A(50) = 30e^{-0.00369(100)} = 20.7$ g $\checkmark$