MATH 120 EXAM #3 KEY (Spring '09)

1 $x^2+11x+y^2-5y=0 \Rightarrow$
$(x^2+11x+{}^{121}/_4)+(y^2-5y+{}^{25}/_4)=\frac{121}{4}+\frac{25}{4} \Rightarrow$
$(x+{}^{11}/_2)^2+(y-{}^{5}/_2)^2=\frac{146}{4}$, so we've a circle w/ center $(-{}^{11}/_2, {}^{5}/_2)$ & radius $\sqrt{\frac{146}{4}}=\frac{\sqrt{146}}{2}$

2 $y-2=\frac{5}{3}(x+9) \Rightarrow y=\frac{5}{3}x+17$

3 $2y-4x=10 \Rightarrow y-2x=5 \Rightarrow y=2x+5 \Rightarrow$ lines have slope $m=2$. Now, line in question has eq. $y+12=2(x+8) \Rightarrow y=2x+4 \Rightarrow$
$2x-y=-4$

4 $3x-y=-18 \Rightarrow y=3x+18 \Rightarrow$ line has slope 3. So line in question has slope $-{}^1/_3$, yielding eq.
$y-0=-\frac{1}{3}(x-3) \Rightarrow y=-\frac{1}{3}x+1$.

5a $f(-2)=|-2+12|-5=5$

5b $g(0)=\frac{0^2-16}{2\cdot 0+8}=-2$

5c $\mathrm{Dom}(g)=\{x \mid 2x+8\neq 0\}=\{x \mid x\neq -4\}$
$=(-\infty,-4)\cup(-4,\infty)$ ✓
$\mathrm{Ran}(f)=[-5,\infty)$ since $|x+12|\geq 0 \Rightarrow |x+12|-5\geq -5$
$\Rightarrow f(x)\geq -5$ ✓

6a $\mathrm{Dom}\, f=\{x \mid 36-x^2\geq 0\}=[-6,6]$
$\mathrm{Dom}\, g=\{x \mid x+2\geq 0\}=[-2,\infty)$

6b $(fg)(x)=f(x)g(x)=\sqrt{36-x^2}\cdot\sqrt{x+2}=\sqrt{(36-x^2)(x+2)}$
w/ $\mathrm{Dom}\, fg=[-6,6]\cap[-2,\infty)=[-2,6]$

6c $(f/g)(x)=f(x)/g(x)=\sqrt{\frac{36-x^2}{x+2}}$
w/ $\mathrm{Dom}\, f/g=(-2,6]$

7a $\mathrm{Dom}\, f=[0,\infty)$
$\mathrm{Dom}\, g=\{x \mid 2x-4\geq 0\}=\{x \mid x\geq 2\}=[2,\infty)$

7b $(fog)(x)=f(g(x))=f\left(16-\sqrt{2x-4}\right)$
$=\sqrt{3\left(16-\sqrt{2x-4}\right)}$ w/
$\mathrm{Dom}\, fog=\{x \mid x\in \mathrm{Dom}\, g \ \&\ g(x)\in \mathrm{Dom}\, f\}$
$=\{x \mid x\in[2,\infty) \ \&\ 16-\sqrt{2x-4}\in[0,\infty)\}$, where
$16-\sqrt{2x-4}\geq 0 \Rightarrow 0\leq\sqrt{2x-4}\leq 16 \Rightarrow 0\leq 2x-4\leq 256$
$\Rightarrow 2\leq x\leq 130$. Thus...
$\mathrm{Dom}\, fog=\{x \mid x\in[2,\infty) \ \&\ x\in[2,130]\}=[2,130]$ ✓

7c $(gof)(x)=g(f(x))=g(\sqrt{3x})=16-\sqrt{2\sqrt{3x}-4}$
$\mathrm{Dom}\, gof=[{}^4/_3,\infty)$. ✓

8 Let $g(x)=4-3x$ & $f(x)=x^{-10}$

9 Note $h(2)=0$ & $h(-1)=0$

10 $p^{-1}(x)=\frac{9-x}{2}$ ✓

11 Let $y=\frac{x-1}{x+2}$, then $y(x+2)=x-1 \Rightarrow$
$xy+2y=x-1 \Rightarrow xy-x=-2y-1 \Rightarrow$
$x=-\frac{2y+1}{y-1} \Rightarrow q^{-1}(x)=-\frac{2x+1}{x-1}$ ✓