MATH 120 EXAM #4 KEY (FALL 2023)

1a $D_f = \{x \,|\, 4 - 7x > 0\} = (-\infty, \frac{4}{7})$.

1b $D_f = \left\{ x \,\middle|\, \dfrac{x+3}{x^2-9} > 0 \right\} = \left\{ x \,\middle|\, \dfrac{1}{x-3} > 0 \text{ and } x \neq -3 \right\} = (3, \infty)$.

2 $\log \dfrac{x(x^2-1)}{7(x+1)} = \log \dfrac{x(x-1)}{7}$.

3a Get $8^{1-2x} = 8^{2(x-4)}$, which implies $1 - 2x = 2(x-4)$, and so $x = \frac{9}{4}$.

3b Let $u = e^x$ to get $u^2 - 3u + 2 = 0$, and thus $u = 1$ or $u = 2$. Now, $e^x = 1$ has solution $x = 0$, and $e^x = 2$ has solution $x = \ln 2$. Solution set: $\{0, \ln 2\}$.

3c Write $\ln(3 - x) = 2$, which is equivalent to $e^2 = 3 - x$, and hence $x = 3 - e^2$.

3d Consolidate to get $\log_9(x-5)(x+3) = 1$, which is equivalent to $9^1 = (x-5)(x+3)$, and so $x^2 - 2x - 24 = 0$. Solving the quadratic equation yields $x = -4, 6$; however, -4 is an extraneous solution for the original logarithmic equation. Solution set: $\{6\}$.

4 For $A(t) = 50e^{-kt}$ we have $\frac{1}{2} \cdot 50 = A(25) = 50e^{-25k}$, so $e^{-25k} = \frac{1}{2}$, and hence $k = 0.02773$. The completed model is now $A(t) = 50e^{-0.02773t}$, and we find t such that $A(t) = 32$. This implies

$$50e^{-0.02773t} = 32,$$

or $e^{-0.02773t} = 0.64$. Solving, we get $t \approx 16.1$ years.

5 The model will have the form $A(t) = A_0e^{-kt}$. Given is that $A(5730) = \frac{1}{2}A_0$. Thus $A_0e^{-5730k} = \frac{1}{2}A_0$, which becomes $e^{-5730k} = \frac{1}{2}$, and hence $k = \frac{\ln 2}{5730} \approx 1.210 \times 10^{-4}$. The model is now $A(t) = A_0e^{-0.0001210t}$.

We now find time t for which $A(t) = 0.71A_0$, or $A_0e^{-0.0001210t} = 0.71A_0$. Solving, we have $e^{-0.0001210t} = 0.71$, giving $-0.0001210t = \ln 0.71$, and finally $t \approx 2830$. That is, the artifact was made around 2830 years ago.

6 Solution is $(-6, -2)$.

7 Note from the first two equations that $x + y + 6z = 3 = x + y + 3z$, so $6z = 3z$, and hence $z = 0$. This immediately reduces the system to two variables, and solving yields $x = -1$ and $y = 4$. Solution is $(-1, 4, 0)$.

8 Letting x be the number of adults and y the number of children, we obtain the following system:

$$\begin{cases} x + y = 325 \\ 9x + 7y = 2495 \end{cases}$$

Solving yields $x = 110$ and $y = 215$; that is, 110 adults and 215 children went to the clown circus.