Math 120 Exam #3 Key (Fall 2023)

1 Complete squares to get $(x-2)^2 + (y+1)^2 = 9$. Center is at $(2,-1)$, radius is 3.

2a It's at the midpoint between the given points, so use the midpoint formula to get $(4,5)$.

2b It's the distance between the center $(4,5)$ and either of the given points. Use the distance formula to get $r = \sqrt{2}$.

2c With center $(4,5)$ and radius $\sqrt{2}$ the equation is $(x-4)^2 + (y-5)^2 = 2$.

3 With $-\frac{b}{2a} = \frac{5}{4}$, vertex is at $(\frac{5}{4}, f(\frac{5}{4})) = (\frac{5}{4}, -\frac{73}{8})$. The domain is $(-\infty,\infty)$ and the range is $[-\frac{73}{8},\infty)$.

4 Have $f(x) = a(x-h)^2 + k$ with $(h,k) = (-3,-1)$, so $f(x) = a(x+3)^2 - 1$. Now use the fact that $f(-2) = -3$ to find that $a = -2$. Therefore $f(x) = -2(x+3)^2 - 1$.

5 From the long division

$$\begin{array}{r|l} & x^2 + x + 1 \\ \hline x^2 - x + 2 & x^4 \qquad + 2x^2 - 5x - 16 \\ & -x^4 + x^3 - 2x^2 \\ \hline & \qquad x^3 \qquad - 5x \\ & \qquad -x^3 + x^2 - 2x \\ \hline & \qquad\qquad x^2 - 7x - 16 \\ & \qquad\qquad -x^2 + x - 2 \\ \hline & \qquad\qquad\quad -6x - 18 \end{array}$$

we have

$$\frac{x^4 + 2x^3 - 4x^2 - 5x - 6}{x^2 - x + 2} = x^2 + x + 1 - \frac{6x + 18}{x^2 - x + 2}.$$

6 The model is $f(x) = C(x+2)[x-(3-i)][x-(3+i)]$, where $3+i$ must also be a zero in order to have real coefficients. Expanding yields $f(x) = C(x^3 - 4x^2 - 2x + 20)$, and to satisfy $f(1) = -24$ we must have $C = -\frac{8}{5}$. Therefore

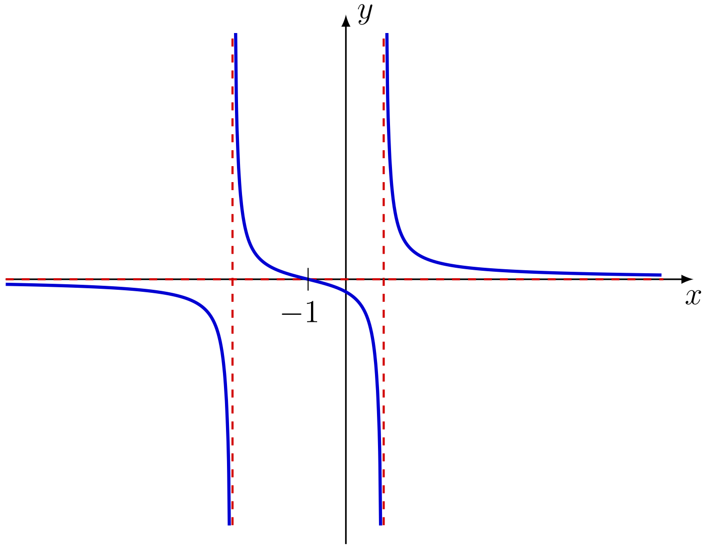
$$f(x) = -\frac{8}{5}x^3 + \frac{32}{5}x^2 + \frac{16}{5}x - 32.$$

7 With $f(x) = x^4 - x^3 + 2x^2 - 4x - 8$, equation is $f(x) = 0$. Possible rational zeros of f are $\pm1, \pm2, \pm4, \pm8$. Through trial-and-error we find 2 is a zero of f, so that $x - 2$ is a factor of $f(x)$, and with synthetic division we obtain $f(x) \div (x-2) = x^3 + x^2 + 4x + 4$. Now

$$f(x) = (x-2)(x^3 + x^2 + 4x + 4) = (x-2)[x^2(x+1) + 4(x+1)] = (x-2)(x+1)(x^2+4).$$

From this factorization we obtain the zeros of f, which are also the solutions to the given equation: $\{2, -1, 2i, -2i\}$.

8 (1) $D_R = \{x \mid x \neq -3, 1\}$; (2) No symmetry; (3) x-intercept is -1, and y-intercept is $R(0) = -\frac{1}{3}$; (4) v.a. are $x = -3$ and $x = 1$; (5) h.a. is $y = 0$; (6) It's helpful to get, say, $R(-4) = -\frac{3}{5}$, $R(-2) = \frac{1}{3}$, $R(2) = \frac{3}{5}$ to fully ascertain where the graph is above the x-axis or below it. For (7) the sketch should resemble the graph below.



9a Write as $(x-4)(x+3) < 0$. Solution set is $(-3, 4)$.

9b Get 0 on one side and a single quotient on the other:

$$\frac{2x+1}{x-3} - 3 \leq 0 \quad \hookrightarrow \quad \frac{10-x}{x-3} \leq 0.$$

Let $f(x) = \frac{10-x}{x-3}$, so inequality is $f(x) \leq 0$. Now, $f(x) = 0$ only if $x = 10$, and $f(x)$ is undefined only if $x = 3$. Use 3 and 10 to partition the real line into subintervals $(-\infty, 3)$, $(3, 10)$, and $(10, \infty)$. Pick a test value in each subinterval to find where $f(x) < 0$ using the IVT. Knowing where $f(x) < 0$ and where $f(x) = 0$ solves $f(x) \leq 0$. Solution set is $(-\infty, 3) \cup [10, \infty)$.