MATH 120 EXAM #2 KEY (FALL 2023)

1 $f(-3) = -\frac{9}{2}$, $f(-x) = \dfrac{x^2}{1-x}$, $f(x+1) = \dfrac{(x+1)^2}{x+2}$.

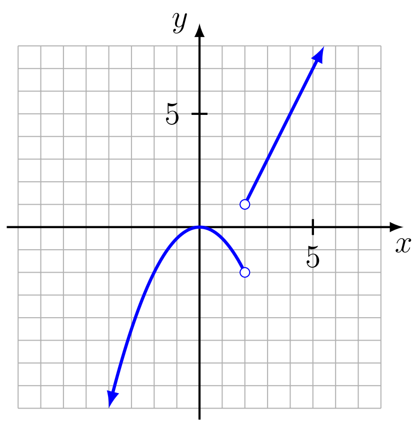
2a $h(-1) = 0$, $h(2)$ is undefined, $h(4) = 4$.

2b $D_g = (-3, -1] \cup [0, 2) \cup (2, 4]$, $R_g = [0, 1) \cup [2, 4]$.

3a Symmetric about origin only.

3b $R(-x) = \dfrac{(-x)^4 - 2(-x)^2 + 3}{(-x)^3} = -\dfrac{x^4 - 2x^2 + 3}{x^3} = -R(x)$, so R is odd.

4a



4b The graph may help: $D_q = (-\infty, 2) \cup (2, \infty)$, $R_q = (-\infty, 0] \cup (1, \infty)$.

5 Slope is $\frac{8}{11}$, so $y - 1 = \frac{8}{11}(x+3)$ is the equation, which becomes $y = \frac{8}{11}x + \frac{35}{11}$.

6 Equation is $y - (-6) = -\frac{3}{2}(x-2)$, which in slope-intercept form is $y = -\frac{3}{2}x - 3$. The y-intercept is -3.

7 $y - 2x + 5 = 0$ becomes $y = 2x - 5$, so the given line has slope 2, and hence L has slope $-\frac{1}{2}$. Equation for L is thus $y = -\frac{1}{2}x - 3$.

8a $D_f = (-\infty, -7) \cup (-7, 7) \cup (7, \infty)$.

8b $D_r = \{x \mid x \neq 0 \text{ and } 15/x \neq 5\} = (-\infty, 0) \cup (0, 3) \cup (3, \infty)$.

9a $D_F = [2, \infty)$, $D_G = [-\frac{5}{2}, \infty)$.

9b $(F - G)(x) = \sqrt{x - 2} - \sqrt{2x + 5}$ with $D_{F-G} = D_F \cap D_G = [2, \infty)$.

9c $(F/G)(x) = \dfrac{\sqrt{x - 2}}{\sqrt{2x + 5}}$ with $D_{F/G} = [2, \infty)$.

10a $(f \circ g)(x) = f(g(x)) = \sqrt{\dfrac{5}{x - 4}}$.

10b $D_f = [0, \infty)$ and $D_g = \{x \,|\, x \neq 4\}$, so

$$\begin{aligned} D_{f \circ g} &= \{x \,|\, x \in D_g \text{ and } g(x) \in D_f\} \\ &= \{x \,|\, x \neq 4 \text{ and } \tfrac{5}{x-4} \geq 0\} \\ &= \{x \,|\, x > 4\} = (4, \infty). \end{aligned}$$

11a Set $y = f(x)$, and solve for x:

$$y = \frac{7 - 3x}{3x + 2} \quad \hookrightarrow \quad 3xy + 2y = 7 - 3x \quad \hookrightarrow \quad x = \frac{7 - 2y}{3y + 3} \quad \hookrightarrow \quad f^{-1}(y) = \frac{7 - 2y}{3y + 3}.$$

11b $R_{f^{-1}} = D_f = (-\infty, -\frac{2}{3}) \cup (-\frac{2}{3}, \infty)$ and $R_f = D_{f^{-1}} = (-\infty, -1) \cup (-1, \infty)$.