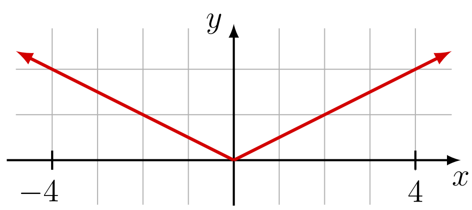
Math 120 Exam #1 Key (Fall 2023)

1a Quadrants I, III.

1b Both inequalities are only happy simultaneously in quadrant III.

2



3a Get $7x - 2 = 4x - 5$, and then $x = -1$.

3b Multiply by $(x+3)(x-2)$ to get

$$6(x-2) + 20 = 5(x+3) \quad \Rightarrow \quad x = 7.$$

Solution set is $\{7\}$.

4 Let x be the amount invested at 2.19%, so $\$30,000 - x$ is the amount invested at 2.45%. Then $0.0219x + 0.0245(30,000 - x) = 705.88$, which solves to give $x = \$11,200$. So \$11,200 was invested at 2.19% and \$18,800 was invested at 2.45%.

5 $pm = T - D$, and so $p = \dfrac{T-D}{m}$.

6a FOIL yields $18 + 30i - 3i - 5i^2 = 23 + 27i$.

6b $\dfrac{4i}{1+i} \cdot \dfrac{1-i}{1-i} = \dfrac{4i - 4i^2}{1 - i^2} = \dfrac{4+4i}{2} = 2 + 2i$.

7 See the long division work below. Since the remainder is 1, we have $i^{613} = i^1 = i$.

$$\begin{array}{r} 153 \\ 4\,\overline{)\,613} \\ \underline{4} \\ 21 \\ \underline{20} \\ 13 \\ \underline{12} \\ 1 \end{array}$$

8a $x = \dfrac{-1 \pm \sqrt{1^2 - 4(5)(-2)}}{2(5)} = \dfrac{-1 \pm \sqrt{41}}{10}$.

8b $x^2 - 6x = -10 \hookrightarrow x^2 - 6x + 9 = -10 + 9 \hookrightarrow (x-3)^2 = -1 \hookrightarrow x - 3 = \pm\sqrt{-1}$, and so we obtain $x = 3 \pm i$.

9 Let x be the length of the sides of the original square. Then $x + 2$ is the length of the sides of the new square, which has area 36, and so $(x+2)^2 = 36$. Now we get $x + 2 = \pm 6$, or $x = -2 \pm 6$, which yields 4 cm as the only physically sensible answer.

10a Write $\sqrt{2x + 15} = x + 6$, so $2x + 15 = (x+6)^2$, which becomes $x^2 + 10x + 21 = 0$. Solving this quadratic equation yields $x = -7, -3$. But -7 is extraneous, so the solution set is $\{-3\}$.

10b Factor: $(2x^{1/3} - 3)(x^{1/3} + 5) = 0$, so $2x^{1/3} = 3$ or $x^{1/3} = -5$, and hence $x = \frac{27}{8}, -125$. (The substitution $u = x^{1/3}$ may help but is not essential.)

10c We get $2x - 1 = \pm 5$, and hence $x = \dfrac{1 \pm 5}{2} = -2, 3$.

11a Write $5x \leq -4$, so that $x \leq -\frac{4}{5}$. Solution set: $(-\infty, -\frac{4}{5}]$.

11b We get $|-2x + 7| > 4$, implying $-2x + 7 > 4$ or $-2x + 7 < -4$, and thus $x < \frac{3}{2}$ or $x > \frac{11}{2}$. Solution set is $(-\infty, \frac{3}{2}) \cup (\frac{11}{2}, \infty)$.

11c Divide by -3 to get $|x + 7| \leq 9$, so $-9 \leq x + 7 \leq 9$, and therefore $-16 \leq x \leq 2$. Solution set is $[-16, 2]$.