MATH 120 EXAM #4 KEY (FALL 2022)

1a $D_f = \{x \,|\, 4-6x>0\} = (-\infty, \frac{2}{3})$.

1b $D_f = \left\{x \,\middle|\, \dfrac{x+3}{8-x} > 0\right\} = (-3, 8)$.

2 $\log \dfrac{x(x^2-1)}{7(x+1)} = \log \dfrac{x(x-1)}{7}$.

3 $\log_b \sqrt[3]{\dfrac{25}{16}} = \dfrac{1}{3}\log_b\left(\dfrac{25}{16}\right) = \dfrac{2}{3}\log_b\left(\dfrac{5}{4}\right) = \dfrac{2}{3}(\log_b 5 - 2\log_b 2) = \dfrac{2}{3}(\beta - 2\alpha)$.

4a $2^{3(1-x)} = 2^{2(x+2)}$ implies $3(1-x) = 2x+4$, and so $x = -\frac{1}{5}$.

4b Let $u = 2^x$ to get $u^2 + u - 12 = 0$, and thus $u = -4$ or $u = 3$. Now, $2^x = -4$ has no solution, but $2^x = 3$ gives $x = \frac{\ln 3}{\ln 2}$.

4c Write $\log_3(x+6)(x+4) = 1$, so $(x+6)(x+4) = 3$ and hence $x = -7, -3$. But $x = -7$ is extraneous. Solution set is $\{-3\}$.

4d $|\ln x| = 4$ implies $\ln x = \pm 4$, and hence $x = e^{\pm 4}$.

5 For $A(t) = A_0 e^{-kt}$ we have $\frac{1}{2}A_0 = A(7340) = A_0 e^{-7340k}$, so $e^{-7340k} = \frac{1}{2}$, and hence $k = 0.00009443$. The completed model is now $A(t) = A_0 e^{-0.00009443t}$, and we find t such that $A(t) = 0.01A_0$. This implies

$$A_0 e^{-0.00009443t} = 0.01A_0,$$

or $e^{-0.00009443t} = 0.01$. Solving, we get $t \approx 48{,}768$ years.

6 Growth rate is 0.82%. As for the doubling time, it is given by $(\ln 2)/k = (\ln 2)/0.0082 = 84.5$ years.

7 Solution is $(-6, -2)$.

8 Letting x be the first number and y the second number, we obtain the system

$$\begin{cases} 3x + 2y = 8 \\ 2x - y = 3 \end{cases}$$

Solving yields $x = 2$ and $y = 1$.

9 Adding the last two equations gives $4x + 3y = 4$. Use this and the first equation to solve for x and y, and finally get the solution $(x, y, z) = (1, 0, -3)$.