MATH 120 EXAM #3 KEY (FALL 2022)

1 $\sqrt{(-\frac{1}{2}-7)^2+(3+5)^2}=\sqrt{481/4}=\dfrac{\sqrt{481}}{2}$.

2 Complete squares to get $(x-2)^2+(y-6)^2=49$. Center is at $(2,6)$, radius is 7.

3 With $-\frac{b}{2a}=-3$, vertex is at $(-3,f(-3))=(-3,21)$. The domain is $(-\infty,\infty)$ and the range is $(-\infty,21]$.

4a Rocket reaches maximum height at time $t=-\frac{b}{2a}=4$ seconds.

4b Maximum height is $h(4)=256$ feet.

4c Find times t for which $h(t)=0$, or $-16t^2+128t=0$. Solutions are $t=0$ and $t=8$. So rocket returns to Earth at 8 seconds.

5 From the long division

$$
\begin{array}{rr}
 & x^2 \qquad\qquad +2 \\
\cline{2-2}
x^2+2x-2\,\big) & x^4+2x^3 \qquad\quad -5x-6 \\
 & -x^4-2x^3+2x^2 \qquad\qquad \\
\cline{2-2}
 & 2x^2-5x-6 \\
 & -2x^2-4x+4 \\
\cline{2-2}
 & -9x-2
\end{array}
$$

we have

$$\frac{x^4+2x^3-4x^2-5x-6}{x^2+x-2}=x^2+2-\frac{9x+2}{x^2+2x-2}.$$

6 Equation is $f(x)=0$ with $f(x)=x^4-2x^2-16x-15$. Possible rational zeros are $\pm1,\pm3,\pm5,\pm15$. Through trial-and error we find -1 is a hit:

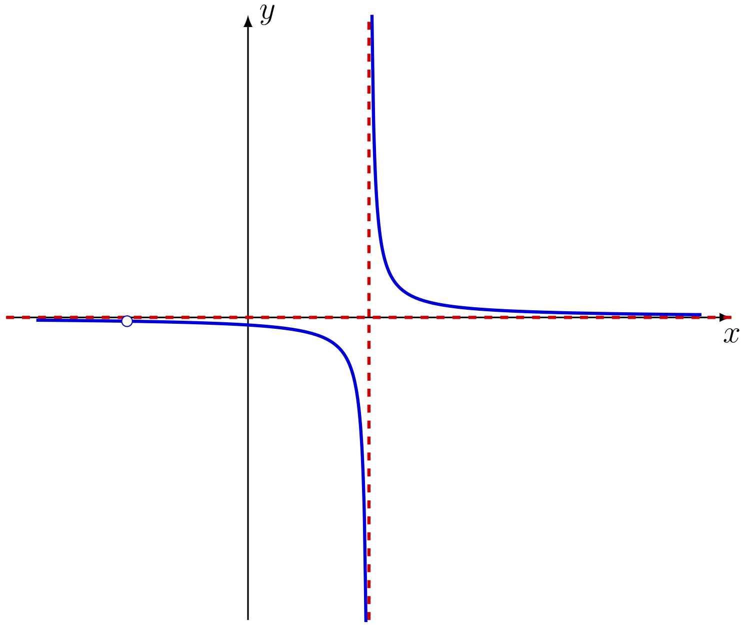
$$
\begin{array}{r|rrrr|r}
-1 & 1 & 0 & -2 & -16 & -15 \\
 & & -1 & 1 & 1 & 15 \\
\hline
 & 1 & -1 & -1 & -15 & 0
\end{array}
\quad\longrightarrow\quad f(x)=(x+1)(x^3-x^2-x-15).
$$

The factor $g(x)=x^3-x^2-x-15$ has the same list of possible rational zeros, and we find 3 is one:

$$
\begin{array}{r|rrr|r}
3 & 1 & -1 & -1 & -15 \\
 & & 3 & 6 & 15 \\
\hline
 & 1 & 2 & 5 & 0
\end{array}
\quad\longrightarrow\quad g(x)=(x-3)(x^2+2x+5).
$$

Now $f(x)=(x+1)(x-3)(x^2+2x+5)$, so $f(x)=0$ if and only if $x=-1,3,-1\pm2i$.

7 (1) $D_R = \{x \mid x \neq \pm 4\}$; (2) No symmetry; (3) y-intercept $-\frac{1}{4}$; (4) v.a. is $x = 4$, hole at $(-4, -\frac{1}{8})$; (5) h.a. is $y = 0$; (6) additional points as needed. For (7) the sketch should resemble the graph below.



8a Let $f(x) = (x+1)(x-2)(x+3)$, so inequality is $f(x) > 0$. The x-intercepts are $-3, -1, 2$. For intervals $(-\infty, -3)$, $(-3, -1)$, $(-1, 2)$, $(2, \infty)$ choose test values like -4, -2, 0, 3, respectively. Since $f(-4) < 0$, $f(-2) > 0$, $f(0) < 0$, $f(3) > 0$, by the Intermediate Value Theorem we conclude that $f(x) > 0$ on $(-3, -1) \cup (2, \infty)$.

8b Write as

$$\frac{2}{x-9} - 1 \geq 0 \quad \hookrightarrow \quad \frac{11-x}{x-9} \geq 0.$$

Now, $f(x) = \dfrac{11-x}{x-9} = 0$ only when $x = 11$, so we now consider $f(x) > 0$. Since f has zero 11 and vertical asymptote $x = 9$, we choose test values in intervals $(-\infty, 9)$, $(9, 11)$, and $(11, \infty)$, and use the Intermediate Value Theorem to conclude that $f(x) > 0$ only when $x \in (9, 11)$. This is the solution set for $f(x) > 0$, and thus the solution set for $f(x) \geq 0$ (equivalent to the original inequality) is $(9, 11]$.