Math 120 Exam #3 Key (Fall 2021)

1 $\sqrt{(-4-2)^2+(-1-(-3))^2}=\sqrt{40}=2\sqrt{10}$.

2 $(x-2)^2+(y+1)^2=9$ is standard form. Center is at $(2,-1)$, radius is 3.

3 With $-\frac{b}{2a}=2$, vertex is at $(2,f(2))=(2,-11)$. Domain is $(-\infty,\infty)$, range is $[-11,\infty)$.

4a Rocket reaches maximum height at time $t=-\frac{b}{2a}=4$ seconds.

4b Maximum height is $h(4)=256$ feet.

4c Find times t for which $h(t)=0$, or $-16t^2+128t=0$. Solutions are $t=0$ and $t=8$. So rocket returns to Earth at 8 seconds.

5 From the long division

$$
\begin{array}{r|rrrrr}
 & & x^2 & +x & -3 & \\
\hline
x^2+x-2 & x^4 & +2x^3 & -4x^2 & -5x & -6 \\
 & -x^4 & -x^3 & +2x^2 & & \\
\hline
 & & x^3 & -2x^2 & -5x & \\
 & & -x^3 & -x^2 & +2x & \\
\hline
 & & & -3x^2 & -3x & -6 \\
 & & & 3x^2 & +3x & -6 \\
\hline
 & & & & & -12
\end{array}
$$

we have

$$\frac{x^4+2x^3-4x^2-5x-6}{x^2+x-2}=x^2+x-3-\frac{12}{x^2+x-2}.$$

6 Equation is $f(x)=0$ with $f(x)=x^4-2x^2-16x-15$. Possible rational zeros are $\pm1,\pm3,\pm5,\pm15$. Through trial-and error we find -1 is a hit:

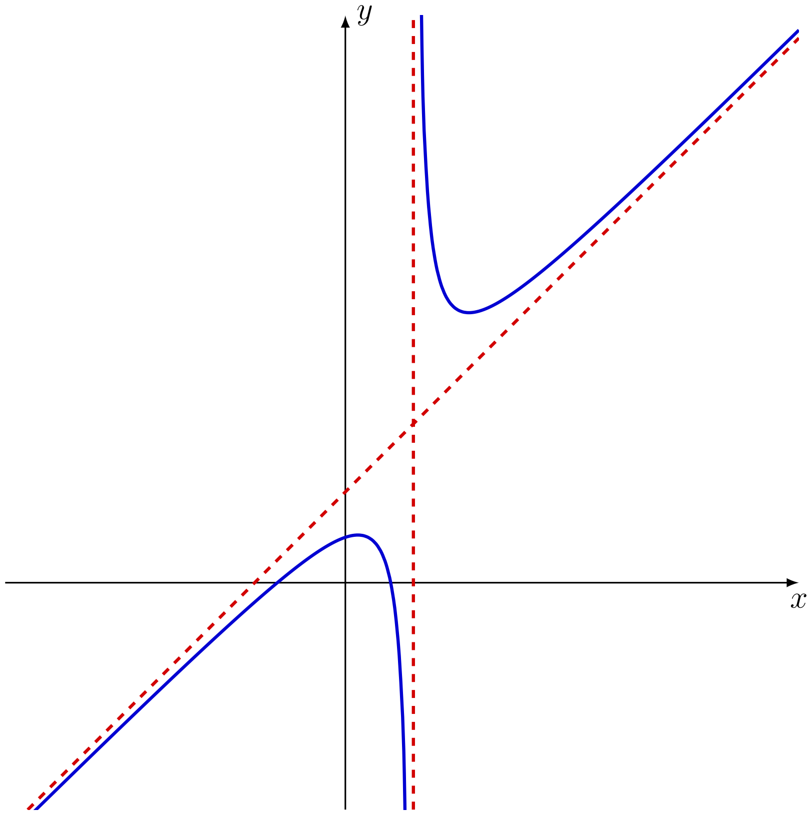
$$
\begin{array}{r|rrrr|r}
-1 & 1 & 0 & -2 & -16 & -15 \\
 & & -1 & 1 & 1 & 15 \\
\hline
 & 1 & -1 & -1 & -15 & 0
\end{array}
\quad\longrightarrow\quad f(x)=(x+1)(x^3-x^2-x-15).
$$

The factor $g(x)=x^3-x^2-x-15$ has the same list of possible rational zeros, and we find 3 is one:

$$
\begin{array}{r|rrr|r}
3 & 1 & -1 & -1 & -15 \\
 & & 3 & 6 & 15 \\
\hline
 & 1 & 2 & 5 & 0
\end{array}
\quad\longrightarrow\quad g(x)=(x-3)(x^2+2x+5).
$$

Now $f(x)=(x+1)(x-3)(x^2+2x+5)$, so $f(x)=0$ if and only if $x=-1,3,-1\pm2i$.

7 (1) No symmetry; (2) $R(0) = 2$; (3) $R(x) = 0$ if $x = -3, 2$; (4) v.a. is $x = 3$; (5) $y = x + 4$ is slant asymptote; (6) additional points as needed. For (7) the sketch should resemble the graph below.



8a Let $f(x) = (x + 1)(x - 2)(x + 3)$, so inequality is $f(x) > 0$. The x-intercepts are $-3, -1, 2$. For intervals $(-\infty, -3)$, $(-3, -1)$, $(-1, 2)$, $(2, \infty)$ choose test values like -4, -2, 0, 3, respectively. Since $f(-4) < 0$, $f(-2) > 0$, $f(0) < 0$, $f(3) > 0$, by the Intermediate Value Theorem we conclude that $f(x) > 0$ on $(-3, -1) \cup (2, \infty)$.

8b Write as

$$\frac{x}{x-6} - 1 \leq 0 \quad \hookrightarrow \quad \frac{6}{x-6} \leq 0.$$

Now, $\frac{6}{x-6} = 0$ has no solution, while $\frac{6}{x-6} < 0$ can only occur if $x - 6 < 0$, or $x < 6$. Solution set is therefore $(-\infty, 6)$.