## Math 120 Exam \#1 Key (Fall 2021)

1a Quadrants III, IV

1b Quadrant IV

2


3a Multiply by $2(x+3)(x-2)$ to get

$$
6(x-2)=5(x-2)+2(x+3) \Rightarrow 6 x-12=7 x-4 \Rightarrow x=-8
$$

3b Equation becomes $4 x+7=4 x+7$, which is satisfied for any real $x$, and so the solution set is $\mathbb{R}$ (the set of real numbers).

4 Say $x$ was invested at $15 \%$, so that $15,000-x$ was invested at $-7 \%$, and then

$$
0.15 x-0.07(15,000-x)=1590
$$

Solving gives $x=12,000$, so $\$ 12,000$ was invested at $15 \%$ and $\$ 3000$ was invested at $-7 \%$.
$5 b=\frac{2 A}{h}-a$.

6a FOIL procedure gives $21-20 i$.
$\mathbf{6 b} \frac{2+4 i}{2-i} \cdot \frac{2+i}{2+i}=\frac{4+10 i+4 i^{2}}{4-i^{2}}=\frac{10 i}{5}=2 i$.

7 The division 513/4 has remainder 1, and so $i^{513}=i^{1}=i$.

8a Factors as $(4 x-1)(x-3)=0$, so that $x=\frac{1}{4}, 3$.

8b We get $x^{2}-2 x+1=\frac{3}{2}$, or $(x-1)^{2}=\frac{3}{2}$, so that $x=1 \pm \sqrt{\frac{3}{2}}=1 \pm \frac{\sqrt{6}}{2}$.
$8 \mathbf{c} \quad x=\frac{2 \pm \sqrt{(-2)^{2}-4(1)(17)}}{2(1)}=\frac{2 \pm 8 i}{2}=1 \pm 4 i$.

9 Let the original sheet metal have sides of length $x$. Referring to the figure below, the volume $V$ of the resultant box will be $3(x-6)^{2}$, which must equal 80 , and so $3(x-6)^{2}=80$. Solving this equation leads to $x=6 \pm \sqrt{\frac{80}{3}}$. Since $x=6-\sqrt{\frac{80}{3}}$ results in $x-6$ being negative, we must have $x=6+\sqrt{\frac{80}{3}}$. The dimensions of the box are thus $\sqrt{\frac{80}{3}} \mathrm{~cm} \times \sqrt{\frac{80}{3}} \mathrm{~cm} \times 3 \mathrm{~cm}$.


10a Write $\sqrt{2 x-3}=1+\sqrt{x-2}$, square to get $2 x-3=1+2 \sqrt{x-2}+(x-2)$, and then isolate the remaining radical to get

$$
2 \sqrt{x-2}=x-2 \Rightarrow 4(x-2)=(x-2)^{2} \Rightarrow x^{2}-8 x+12=0
$$

The trinomial factors, giving $(x-6)(x-2)=0$, and therefore $x=2,6$.

10b Factor: $\left(2 x^{1 / 3}-3\right)\left(x^{1 / 3}+5\right)=0$, so $2 x^{1 / 3}=3$ or $x^{1 / 3}=5$, and hence $x=\frac{27}{8},-125$. (The substitution $u=x^{1 / 3}$ may help but is not essential.)

10c We get $|x+1|=-4$, which is impossible and so the solution set is $\varnothing$.

11a Solving leads to $x \geq 8$, so the solution set is $[8, \infty)$.

11b We get $|2 x+2|<8$, implying $-8<2 x+2<8$, and finally $-5<x<3$. Solution set is $(-5,3)$.

11c Divide by -3 to get $|x+7| \leq 9$, so $-9 \leq x+7 \leq 9$, and therefore $-16 \leq x \leq 2$. Solution set is $[-16,2]$.

