Math 120 Exam #3 Key (Fall 2014)

1 Solve $-6x + 2y = 3$ for y to get $y = 3x + \frac{3}{2}$, which shows the slope for the line given by $-6x + 2y = 3$ to be 3. Since the line L through $(5, -3)$ is parallel to this line, its slope must also be 3. So the equation for L is $y - (-3) = 3(x - 5)$, or

$$y = 3x - 18.$$

2 Solve $4x - 3y = 6$ for y to get $y = \frac{4}{3}x - 2$, which shows the slope for the line given by $4x - 3y = 6$ to be $\frac{4}{3}$. Since the line L through $(1, 9)$ is perpendicular to this line, its slope must be $-\frac{3}{4}$. So the equation for L is $y - 9 = -\frac{3}{4}(x - 1)$, or

$$y = -\tfrac{3}{4}x + \tfrac{39}{4}.$$

3 Domain is $(-\infty, \infty)$ and range is $(-\infty, \infty)$. The relation is not a function.

4a Domain is $[0, \infty)$ and range is $[3, \infty)$

4b Domain is $(-\infty, \infty)$ and range is $[-25, \infty)$

5a $\text{Dom}(\varphi) = (-\infty, 4) \cup (4, \infty)$

5b $\text{Dom}(\omega) = \{x : x + 4 \geq 0\} = \{x : x \geq -4\} = [-4, \infty)$

5c $\text{Dom}(\psi) = \{x : 81 - x^2 \geq 0\} = \{x : x^2 \leq 81\} = \{x : -9 \leq x \leq 9\} = [-9, 9]$

6a By definition

$$(\varphi + \psi)(x) = \varphi(x) + \psi(x) = \frac{x+1}{4-x} + \sqrt{81 - x^2},$$

and

$$\text{Dom}(\varphi + \psi) = \text{Dom}(\varphi) \cap \text{Dom}(\psi) = \big[(-\infty, 4) \cup (4, \infty)\big] \cap [-9, 9] = [-9, 4) \cup (4, 9].$$

6b By definition

$$(\varphi/\omega)(x) = \varphi(x)/\omega(x) = \frac{x+1}{4-x} \cdot \frac{1}{\sqrt{x+4}} = \frac{x+1}{(4-x)\sqrt{x+4}},$$

and

$$\begin{aligned} \text{Dom}(\varphi/\omega) &= \{x : x \in \text{Dom}(\varphi) \cap \text{Dom}(\omega) \text{ and } \omega(x) \neq 0\} \\ &= \{x : x \in (-\infty, 4) \cup (4, \infty) \text{ and } x \in [-4, \infty) \text{ and } x \neq -4\} \\ &= (-4, 4) \cup (4, \infty). \end{aligned}$$

6c By definition

$$(\omega \circ \omega)(x) = \omega(\omega(x)) = \omega\left(\sqrt{x+4}\right) = \sqrt{\sqrt{x+4}+4},$$

and

$$\begin{aligned}\mathrm{Dom}(\omega \circ \omega) &= \{x : x \in \mathrm{Dom}(\omega) \text{ and } \omega(x) \in \mathrm{Dom}(\omega)\} \\ &= \left\{x : x \geq -4 \text{ and } \sqrt{x+4} \geq -4\right\} \\ &= \{x : x \geq -4\} = [-4, \infty).\end{aligned}$$

6d By definition

$$(\omega \circ \psi)(x) = \omega(\psi(x)) = \omega\big(\sqrt{81-x^2}\,\big) = \sqrt{\sqrt{81-x^2}+4},$$

and

$$\begin{aligned}\mathrm{Dom}(\omega \circ \psi) &= \{x : x \in \mathrm{Dom}(\psi) \text{ and } \psi(x) \in \mathrm{Dom}(\omega)\} \\ &= \left\{x : -9 \leq x \leq 9 \text{ and } \sqrt{81-x^2} \geq -4\right\} \\ &= \left\{x : -9 \leq x \leq 9 \text{ and } x^2 \leq 81\right\} \\ &= \{x : -9 \leq x \leq 9\} = [-9, 9].\end{aligned}$$

7 Let $f(x) = \sqrt[5]{x}$ and $g(x) = 2x - 9$. Then

$$(f \circ g)(x) = f(g(x)) = f(2x-9) = \sqrt[5]{2x-9} = H(x).$$

8 Suppose that $f(a) = f(b)$. Then

$$2a^3 - 1 = 2b^3 - 1 \;\;\Rightarrow\;\; 2a^3 = 2b^3 \;\;\Rightarrow\;\; a^3 = b^3 \;\;\Rightarrow\;\; a = b.$$

Therefore f is one-to-one.

9 Since $g(1) = 1^6 - 12 = -11 = (-1)^6 - 12 = g(-1)$, g is not one-to-one.

10a Suppose that $f(x) = y$. Then

$$y = \frac{x+2}{1-3x} \;\;\Rightarrow\;\; y - 3xy = x + 2 \;\;\Rightarrow\;\; 3xy + x = y - 2 \;\;\Rightarrow\;\; x = \frac{y-2}{3y+1},$$

and since $f^{-1}(y) = x$ by definition, it follows that

$$f^{-1}(y) = \frac{y-2}{3y+1}.$$

10b $\mathrm{Ran}(f) = \mathrm{Dom}(f^{-1}) = (-\infty, -1/3) \cup (-1/3, \infty)$

10c $\mathrm{Ran}(f^{-1}) = \mathrm{Dom}(f) = (-\infty, 1/3) \cup (1/3, \infty)$