Math 120 Exam #1 Key (Fall 2014)

1 $-s^2 + 8t + r^2 = -(-5)^2 + 8(3) + (-4)^2 = -25 + 24 + 16 = 15$

2a $(u^3 - 2u^2 + 5) - 2(-7u^3 + 11u^2) = u^3 - 2u^2 + 5 + 14u^3 - 22u^2 = 15u^3 - 24u^2 + 5$

2b $(3v + 2)(4v^2 - 7v + 6) = 12v^3 - 13v^2 + 4v + 12$

2c $(a - 8b)^2 = a^2 - 16ab + 64b^2$

3 Answer is: $4x^2 + 2x + \dfrac{-7x + 1}{2x^2 - x + 2}$.

$$
\begin{array}{r|l}
2x^2 - x + 2 & 4x^2 + 2x \\
\cline{2-2}
8x^4 + 6x^2 - 3x + 1 & \\
-8x^4 + 4x^3 - 8x^2 & \\
\cline{1-1}
4x^3 - 2x^2 - 3x & \\
-4x^3 + 2x^2 - 4x & \\
\cline{1-1}
-7x &
\end{array}
$$

4a $10ab - 6b + 35a - 21 = 2b(5a - 3) + 7(5a - 3) = (5a - 3)(2b + 7)$

4b $9z^2 + 4z - 2$ is prime

4c $32a^2 + 48ab + 18b^2 = 2(16a^2 + 24ab + 9b^2) = 2(4a + 3b)^2$

4d $36k^2 - 81\ell^4 = 9(4k^2 - 9\ell^4) = 9(2k - 3\ell^2)(2k + 3\ell^2)$

4e $1000x^3 + 343y^3 = (10x)^3 + (7y)^3 = (10x + 7y)(100x^2 - 70xy + 49y^2)$

5a $\dfrac{q^3 + q^2}{7} \cdot \dfrac{49}{q^4 + q^3} = \dfrac{q^2(q+1)}{7} \cdot \dfrac{7^2}{q^3(q+1)} = \dfrac{1}{1} \cdot \dfrac{7}{q} = \dfrac{7}{q}$

5b $\dfrac{x^2 + x - 2}{x^2 + 3x - 4} \div \dfrac{x^2 + 3x + 2}{x^2 + 4x + 3} = \dfrac{(x+2)(x-1)}{(x+4)(x-1)} \cdot \dfrac{(x+3)(x+1)}{(x+2)(x+1)} = \dfrac{x+3}{x+4}$

6 $\dfrac{5}{12x^2y} - \dfrac{7}{6xy^3} = \dfrac{5}{12x^2y} \cdot \dfrac{y^2}{y^2} - \dfrac{7}{6xy^3} \cdot \dfrac{2x}{2x} = \dfrac{5y^2}{12x^2y^3} - \dfrac{14x}{12x^2y^3} = \dfrac{5y^2 - 14x}{12x^2y^3}$

7 We have

$$\frac{1-\dfrac{2}{3x}}{9-\dfrac{4}{x^2}} = \frac{1-\dfrac{2}{3x}}{9-\dfrac{4}{x^2}} \cdot \frac{3x^2}{3x^2} = \frac{3x^2-2x}{27x^2-12} = \frac{x(3x-2)}{3(3x-2)(3x+2)} = \frac{x}{9x+6}$$

8 $\dfrac{(r^{-1/5}s^{2/3})^{15}}{r^{-2}} = \dfrac{r^{-3}s^{10}}{r^{-2}} = \dfrac{s^{10}}{r}$

9 $t^{-5} - 3t^{-3} = t^{-5}(1-3t^2)$.

10a $\sqrt{25j^4k^2} = 5|j^2k| = 5j^2k$, since $j, k > 0$.

10b If $x, z > 0$: $\sqrt{8x^5z^3} = 2x^2z\sqrt{2xz}$. If $x, z < 0$: $\sqrt{8x^5z^3} = 2x^2|z|\sqrt{2xz}$. If either x or z is zero, then the square root is 0. In all other cases the square root is not a real number.

10c $\sqrt[3]{\dfrac{9}{16r^4}} = \dfrac{\sqrt[3]{9}}{\sqrt[3]{16r^4}} = \dfrac{\sqrt[3]{9}}{2r\sqrt[3]{2r}} = \dfrac{\sqrt[3]{9}}{2r\sqrt[3]{2r}} \cdot \dfrac{\sqrt[3]{4r^2}}{\sqrt[3]{4r^2}} = \dfrac{\sqrt[3]{36r^2}}{4r^2}$

10d $\sqrt[4]{\sqrt[3]{12}} = \sqrt[12]{12}$

10e $\sqrt[3]{32} - 5\sqrt[3]{4} + 2\sqrt[3]{108} = 2\sqrt[3]{4} - 5\sqrt[3]{4} + 2 \cdot 3\sqrt[3]{4} = 3\sqrt[3]{4}$

11 $4[2x-(3-x)+5] = -6x-28 \;\Rightarrow\; 4(3x+2) = -6x-28 \;\Rightarrow\; 18x = -36 \;\Rightarrow\; x = -2$

12 $ax+b = 3(x-a) \;\Rightarrow\; ax+b = 3x-3a \;\Rightarrow\; ax+3a = 3x-b \;\Rightarrow\; a(x+3) = 3x-b \;\Rightarrow\; a = \dfrac{3x-b}{x+3}$

13 Let x be the quantity of 60% acid solution to add. We equate the total amount of pure acid present in the two solutions before mixing with the amount of pure acid present in the final mixture:

$$0.60x + 0.45(400) = 0.55(x+400).$$

Solving yields $x = 800$. That is, 800 mL of 60% solution must be added.

14 Let x be the amount of money invested at 5.5%, so that $12,000 - x$ is the amount invested at 4%. Add the interest from each investment to get the total interest of \$560:

$$0.055x + 0.04(12,000 - x) = 560.$$

Solving yields $x = 5,333.33$. That is, \$5,333.33 is invested at 5.5% and \$6,666.67 at 4%.

15a $(3 - 6i) - (-8 - 13i) = 11 + 7i$

15b $(4 - 3i)(2 + 9i) = 8 + 36i - 6i - 27i^2 = 35 + 30i$

15c $\dfrac{3 - 2i}{1 - 2i} \cdot \dfrac{1 + 2i}{1 + 2i} = \dfrac{3 + 6i - 2i - 4i^2}{1 + 2i - 2i - 4i^2} = \dfrac{7 + 4i}{5} = \dfrac{7}{5} + \dfrac{4}{5}i$

15d $i^{265} = i^{4(66)+1} = i^{4(66)} \cdot i^1 = (i^4)^{66} \cdot i = 1^{66} \cdot i = i$