Math 120 Exam #3 Key (Fall 2013)

1 Slope is

$$m = \frac{-1-(-3)}{2-(-5)} = \frac{2}{7}.$$

2 From $2x - y = 1$ we get $y = 2x - 1$, which is a line with slope 2. Thus we need k such that

$$\frac{1-3}{k-2} = -\frac{2}{k-2} = 2.$$

This yields $k = 1$.

3 Domain is $[-3, 4]$ and range is $[-4, 5]$. Relation is not a function since it fails the Vertical Line Test.

4 By direct substitution, $f(2t-1) = -2(2t-1) + 6 = -4t + 8$.

5 We have $\text{Dom}(u) = (-\infty, \infty)$ and $\text{Ran}(u) = [9, \infty)$.

6a We have

$$\text{Dom}(h) = \{x : x - 6 \geq 0\} = \{x : x \geq 6\} = [6, \infty).$$

6b We have

$$\begin{aligned}\text{Dom}(\ell) &= \{x : x - 4 \geq 0 \text{ and } 12 - x \geq 0\} = \{x : x \geq 4 \text{ and } x \leq 12\} \\ &= \{x : 4 \leq x \leq 12\} = [4, 12].\end{aligned}$$

7a We have

$$\text{Dom}(f) = (-\infty, -2) \cup (-2, \infty) \quad \text{and} \quad \text{Dom}(g) = (-\infty, -2) \cup (-2, \infty)$$

7b Finding $f + g$:

$$(f+g)(x) = f(x) + g(x) = \frac{2}{x+2} + \frac{x}{x+2} = \frac{x+2}{x+2} = 1.$$

Now for the domain:

$$\text{Dom}(f+g) = \text{Dom}(f) \cap \text{Dom}(g) = (-\infty, -2) \cup (-2, \infty).$$

7c Finding f/g:

$$(f/g)(x) = f(x)/g(x) = \frac{2}{x+2} \div \frac{x}{x+2} = \frac{2}{x+2} \cdot \frac{x+2}{x} = \frac{2}{x}.$$

Now for the domain:

$$\begin{aligned}\text{Dom}(f/g) &= \{x : x \in \text{Dom}(f) \cap \text{Dom}(g) \text{ and } g(x) \neq 0\} \\ &= \{x : x \neq -2 \text{ and } x \neq 0\} \\ &= (-\infty, -2) \cup (-2, 0) \cup (0, \infty).\end{aligned}$$

7d Finding $f \circ g$:

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{x}{x+2}\right) = \frac{2}{\frac{x}{x+2} + 2} = \frac{2x+4}{3x+4}.$$

Now for the domain:

$$\text{Dom}(f \circ g) = \{x : x \in \text{Dom}(g) \text{ and } g(x) \in \text{Dom}(f)\} = \left\{x : x \neq -2 \text{ and } \frac{x}{x+2} \neq -2\right\},$$

where

$$\frac{x}{x+2} \neq -2 \quad \Leftrightarrow \quad x \neq -2(x+2) \quad \Leftrightarrow \quad x \neq -4/3,$$

and so

$$\text{Dom}(f \circ g) = \{x : x \neq -2 \text{ and } x \neq -4/3\} = (-\infty, -2) \cup \left(-2, -\tfrac{4}{3}\right) \cup \left(-\tfrac{4}{3}, \infty\right)$$

8 Let $f(x) = \sqrt[3]{x}$ and $g(x) = x - 4$. Then

$$(f \circ g)(x) = f(g(x)) = f(x-4) = \sqrt[3]{x-4} = r(x).$$

9 Suppose that $f(a) = f(b)$. Then

$$2a^3 - 1 = 2b^3 - 1 \quad \Rightarrow \quad 2a^3 = 2b^3 \quad \Rightarrow \quad a^3 = b^3 \quad \Rightarrow \quad a = b.$$

Therefore f is one-to-one.

10 Since $g(-2) = 5 = g(2)$, we conclude that g is not one-to-one.

11a Suppose that $f(x) = y$. Then

$$y = \frac{3x+1}{x-3} \quad \Rightarrow \quad xy - 3y = 3x + 1 \quad \Rightarrow \quad xy - 3x = 3y + 1 \quad \Rightarrow \quad x = \frac{3y+1}{y-3},$$

and since $f^{-1}(y) = x$ by definition, it follows that

$$f^{-1}(y) = \frac{3y+1}{y-3}.$$

11b By the definition of f^{-1},

$$\text{Ran}(f) = \text{Dom}(f^{-1}) = (-\infty, 3) \cup (3, \infty)$$

and

$$\text{Ran}(f^{-1}) = \text{Dom}(f) = (-\infty, 3) \cup (3, \infty).$$