Math 120 Exam #2 Key (Fall 2013)

1a From $6x^2+5x-4=0$ we have $(3x+4)(2x-1)=0$, which leads to $x=-4/3$ or $x=1/2$. Solution set: $\{-4/3, 1/2\}$.

1b Divide by 3 to get $x^2-2x=1/3$. Completing the square yields

$$x^2-2x+1=1/3+1,$$

or $(x-1)^2=4/3$. From this comes $x-1=\pm\sqrt{4/3}=\pm 2/\sqrt{3}$, and so the solution set is

$$\left\{1-\frac{2}{\sqrt{3}},\ 1+\frac{2}{\sqrt{3}}\right\}.$$

2 Write as $-16t^2+v_0t+(s_0-h)=0$, so

$$t=\frac{-v_0\pm\sqrt{v_0^2-4(-16)(s_0-h)}}{2(-16)}=\frac{v_0\pm\sqrt{v_0^2+64(s_0-h)}}{32}.$$

3

	Rate	Time	Fraction Done
Inlet	$\frac{1}{7}$	t	$\frac{t}{7}$
Outlet	$-\frac{1}{9}$	t	$-\frac{t}{9}$

If the job is to fill the pool, then the outlet pipe works at a negative rate. Adding the contributions of the two pipes and setting the sum equal to 1 (for 1 pool filled), we obtain the equation

$$\frac{t}{7}-\frac{t}{9}=1.$$

This solves to give $t=\frac{63}{2}=31\frac{1}{2}$ hours.

4 The surface area of the can consists of the areas of the can's circular top and bottom, as well as the can's side which can be bent flat to create a rectangle with width 12 cm and length equal to the circumference of the can. If r is the radius of the can, then the can's top and bottom each have area πr^2. The side has area $(12)(2\pi r)$. Total surface area A is thus

$$A=\pi r^2+\pi r^2+(12)(2\pi r)=2\pi r^2+24\pi r.$$

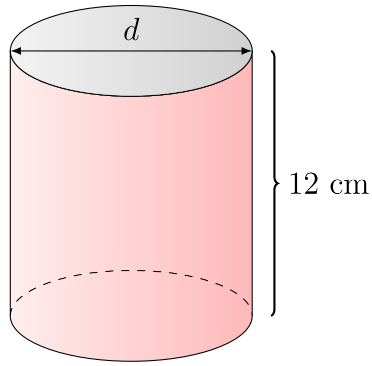
But we're told the surface area is 371 cm^2, so $2\pi r^2+24\pi r=371$. From this we get

$$2\pi r^2+24\pi r-371=0,$$

and so the quadratic formula yields

$$r=\frac{-24\pi\pm\sqrt{(24\pi)^2-4(2\pi)(-371)}}{2(2\pi)}\approx 3.75,\ -15.75.$$

Clearly the radius of the can cannot be negative, so $r=3.75$ cm. The diameter is therefore $d=7.50$ cm



5a Multiply by $x^2(x-1)$ to get $x^2 = 2(x-1)$, which becomes $x^2 - 2x + 2 = 0$. Solving using the quadratic equation (or completing the square), we obtain $x = 1 \pm i$. Solution set: $\{1-i, 1+i\}$.

5b Multiply by $(x-2)(x+2)$ to get $(x+5)(x+2) = 5(x-2)+28$, which becomes $x^2+2x-8 = 0$. Factoring, we get $(x+4)(x-2) = 0$, so that $x = -4, 2$. But 2 is extraneous, so the solution set is $\{-4\}$.

5c We have $\sqrt{5-x} = x-3$. Squaring both sides yields $5 - x = (x-3)^2$, or $x^2 - 5x + 4 = 0$. Factoring, we get $(x-4)(x-1) = 0$, so that $x = 1, 4$. But 1 is extraneous, so the solution set is $\{4\}$.

5d Square both sides to get $(3 - \sqrt{x})^2 = 2\sqrt{x} - 3$, or

$$9 - 6\sqrt{x} + x = 2\sqrt{x} - 3.$$

This becomes $8\sqrt{x} = x + 12$, and squaring again yields $64x = x^2 + 24x + 144$. So we have $x^2 - 40x + 144 = 0$, and factoring results in $(x-36)(x-4) = 0$ and finally $x = 4, 36$. But 36 is extraneous, so the solution set is $\{4\}$.

5e Letting $u = x^2$, the equation becomes quadratic: $3u^2 + 10u - 25 = 0$. Factoring yields $(3u-5)(u+5) = 0$, so $u = 5/3$ or $u = -5$. Replacing u with x^2, we get

$$x^2 = \frac{5}{3} \quad\Rightarrow\quad x = \pm\sqrt{\frac{5}{3}} \quad\Rightarrow\quad x = \pm\frac{\sqrt{15}}{3},$$

and

$$x^2 = -5 \quad\Rightarrow\quad x = \pm\sqrt{-5} \quad\Rightarrow\quad x = \pm i\sqrt{5}.$$

Solution set is

$$\left\{\pm\frac{\sqrt{15}}{3}, \pm i\sqrt{5}\right\}.$$

5f We have either $x - 1 = 11 - 5x$ or $x - 1 = -(11 - 5x)$. The first equation solve to give $x = 2$, and the second equation solves to give $x = 5/2$. Solution set: $\{2, 5/2\}$.

6a Simplifying yields $8x - 3 \leq 3x - 7$, and then $5x \leq -4$, and finally $x \leq -4/5$. Solution set: $(-\infty, -4/5]$.

6b Multiply by 20 to get $-10 < 4(4 - 3x) \leq 5$, whence

$$-10 < 16 - 12x \leq 5 \quad \Rightarrow \quad -26 < -12x \leq -11 \quad \Rightarrow \quad 13/6 > x \geq 11/12.$$

Solution set: $\left[\frac{11}{12}, \frac{13}{6}\right)$.

6c Factoring, we get $(x + 2)(x + 3) < 0$. There are two cases to consider.
Case 1: $x + 2 < 0$ & $x + 3 > 0$. This gives $-3 < x < -2$, which is the interval $(-3, -2)$.
Case 2: $x + 2 > 0$ & $x + 3 < 0$. This gives $x > -2$ & $x < -3$, which is a contradiction.
Solution set: $(-3, -2)$.

6d Manipulate without multiplying by an expression involving x:

$$\frac{x+1}{x-5} \geq 4 \quad \Leftrightarrow \quad \frac{x+1}{x-5} - 4 \geq 0 \quad \Leftrightarrow \quad \frac{x + 1 - 4(x-5)}{x-5} \geq 0 \quad \Leftrightarrow \quad \frac{3(7-x)}{x-5} \geq 0.$$

There are two cases to consider (note that we cannot have $x = 5$ since division by zero would result):
Case 1: $7 - x \geq 0$ & $x - 5 > 0$. This gives $x \leq 7$ & $x > 5$, which is equivalent to $5 < x \leq 7$.
Case 2: $7 - x \leq 0$ & $x - 5 < 0$. This gives $x \geq 7$ & $x < 5$, which is a contradiction.
Solution set: $(5, 7]$.

6e We have

$$|4 - 3x| > 2 \quad \Leftrightarrow \quad 4 - 3x > 2 \quad \text{or} \quad 4 - 3x < -2 \quad \Leftrightarrow \quad x < 2/3 \quad \text{or} \quad x > 2.$$

Solution set: $(-\infty, 2/3) \cup (2, \infty)$.

6f The only way $|x - 3|$ can fail to be greater than 3 is if $x = 3$. Thus the solution set is all real numbers except for 3: $(-\infty, 3) \cup (3, \infty)$.

6g We have

$$|5 - x| \leq 12 \quad \Leftrightarrow \quad -12 \leq 5 - x \leq 12 \quad \Leftrightarrow \quad -17 \leq -x \leq 7 \quad \Leftrightarrow \quad -7 \leq x \leq 17.$$

Solution set: $[-7, 17]$.

7 Complete the square for each variable as follows:

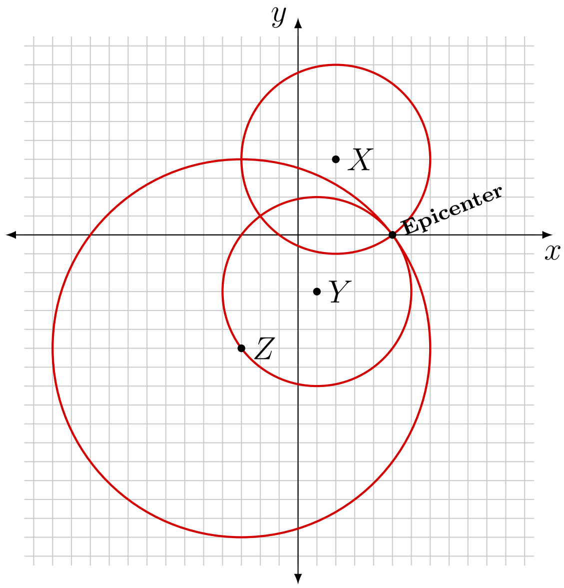
$$\left(x^2 + 5x + \frac{25}{4}\right) + (y^2 - 6y + 9) = 3 + \frac{25}{4} + 9 = \frac{73}{4}.$$

Thus we have

$$\left(x + \frac{5}{2}\right)^2 + (y - 3)^2 = \frac{73}{4},$$

which is a circle with center at $(-5/2, 3)$ and radius $\sqrt{73}/2$.

8 Employ a graphical approach as in the example in the textbook: at coordinates $(2, 4)$ graph a circle of radius 5, at $(1, -3)$ graph a circle of radius 5, and at $(-3, -6)$ graph a circle of radius 10. Looking at the graph below, only the one point $(3, 1)$ lies on all three circles, and so the epicenter of the earthquake must be at $(5, 0)$.



EC The bamboo is 10 feet long, so a right triangle with sides as shown below results. By the Pythagorean theorem we have

$$x^2 + 3^2 = (10 - x)^2.$$

From this we get

$$x^2 + 9 = 100 - 20x + x^2 \quad \Rightarrow \quad 20x = 91 \quad \Rightarrow \quad x = \frac{91}{20} = 4.55 \text{ feet.}$$