MATH 120
FALL 2023
EXAM 3

NAME:

1. [10 pts.] Complete the square and write the equation $x^2 + y^2 - 4x + 2y - 4 = 0$ in the standard form (or center-radius form) for a circle. What is the center and radius of the circle?

2. [5 pts. each] A circle contains points $(3, 6)$ and $(5, 4)$, and the line segment connecting these points contains the center of the circle.
   (a) Find the coordinates of the circle's center.
   (b) Find the radius of the circle.
   (c) Write the standard form (or center-radius form) of the circle's equation.

3. [10 pts.] Find the vertex of the parabola given by $f(x) = 2x^2 - 5x - 6$. In interval notation, what is the domain and range of the function?

4. [10 pts.] A parabola contains the point $(-2, -3)$ and has vertex $(-3, -1)$. Write the equation of the parabola in vertex form.

5. [10 pts.] Divide using long division: $(x^4 + 2x^2 - 5x - 16) \div (x^2 - x + 2)$.

6. [10 pts.] Find a 3rd-degree polynomial function f having real coefficients, zeros -2 and $3 - i$, and such that $f(1) = -24$.

7. [15 pts.] Consider the equation
$$x^4 - x^3 + 2x^2 - 4x - 8 = 0.$$
List all the possible rational roots. Use synthetic division to test the possible rational roots and find actual roots. Then find *all* solutions to the equation, real or complex.

8. [20 pts.] Use the 7-step procedure used in homework to sketch a graph of the rational function
$$R(x) = \frac{x + 1}{x^2 + 2x - 3}.$$
The steps are: (1) Domain; (2) Symmetry; (3) Intercepts; (4) Vertical asymptotes and holes; (5) Horizontal/slant asymptote; (6) Plot additional points as necessary; (7) Graph.

9. [10 pts. each] Solve each inequality, showing use of test values and the Intermediate Value Theorem. Put answers in interval notation.
   (a) $x^2 < x + 12$
   (b) $\dfrac{2x + 1}{x - 3} \leq 3$