Answers to Section 2.7 Supplementary Assignment

$\boxed{1}$ $(f+g)(x) = \sqrt{1-x} + \frac{1}{x-2}$, $(f-g)(x) = \sqrt{1-x} - \frac{1}{x-2}$, $(fg)(x) = \frac{\sqrt{1-x}}{x-2}$, $(f/g)(x) = (x-2)\sqrt{1-x}$

$\mathrm{Dom}\,(f \pm g) = \mathrm{Dom}\,(fg) = (-\infty, 1]$, $\mathrm{Dom}\,(f/g) = (-\infty, 1]$.

$\boxed{2}$ $(f \pm g)(x) = \sqrt{10+x} \pm \sqrt{50-x}$, $(fg)(x) = \sqrt{500+40x-x^2}$, $(f/g)(x) = \sqrt{\frac{10+x}{50-x}}$

$\mathrm{Dom}\,(f \pm g) = \mathrm{Dom}\,(fg) = [-10, 50]$, $\mathrm{Dom}\,(f/g) = [-10, 50)$.

$\boxed{3}$ $(f \pm g)(x) = \sqrt{9-x^2} \pm \sqrt{x^2-1}$, $(fg)(x) = \sqrt{-x^4+10x^2-9}$, $(f/g)(x) = \sqrt{\frac{9-x^2}{x^2-1}}$

$\mathrm{Dom}\,(f \pm g) = \mathrm{Dom}\,(fg) = [-3,-1] \cup [1,3]$, $\mathrm{Dom}\,(f/g) = [-3,-1) \cup (1,3]$

$\boxed{4}$ $(f+g)(x) = 1$, $(f-g)(x) = \frac{2-x}{x+2}$, $(fg)(x) = \frac{2x}{(x+2)^2}$, $(f/g)(x) = 2/x$

$\mathrm{Dom}\,(f \pm g) = \mathrm{Dom}\,(fg) = (-\infty,-2) \cup (-2,\infty)$, $\mathrm{Dom}\,(f/g) = (-\infty,-2) \cup (-2,0) \cup (0,\infty) = \{x \mid x \neq -2, 0\}$

$\boxed{5}$ $(f \pm g)(x) = \frac{1}{\sqrt{2x-3}} \pm (3x^2-8)$, $(fg)(x) = \frac{3x^2-8}{\sqrt{2x-3}}$, $(f/g)(x) = \frac{1}{(3x^2-8)\sqrt{2x-3}}$

$\mathrm{Dom}\,(f \pm g) = \mathrm{Dom}\,(fg) = (3/2, \infty)$, $\mathrm{Dom}\,(f/g) = (3/2, \infty)$

$\boxed{6}$ $(f \pm g)(x) = \sqrt[6]{3-x} \pm \sqrt[4]{x-5}$, $(fg)(x) = \sqrt[6]{3-x} \cdot \sqrt[4]{x-5}$, $(f/g)(x) = \frac{\sqrt[6]{3-x}}{\sqrt[4]{x-5}}$

$\mathrm{Dom}\,(f \pm g) = \mathrm{Dom}\,(fg) = \emptyset$, $\mathrm{Dom}\,(f/g) = \emptyset$.

$\boxed{7}$ $(f \circ g)(x) = f(x+5) = 3(x+5)^2 - 7 = 3x^2+30x+68$

$(g \circ f)(x) = g(3x^2-7) = (3x^2-7)+5 = 3x^2-2$

$(f \circ f)(x) = f(3x^2-7) = 3(3x^2-7)^2-7 = 27x^4-126x^2+140$

$(g \circ g)(x) = g(x+5) = (x+5)+5 = x+10$

$\mathrm{Dom}\,(f \circ g) = \{x \mid x \in \mathrm{Dom}\,g \;\&\; g(x) \in \mathrm{Dom}\,f\} = \{x \mid x \in (-\infty,\infty) \;\&\; x \in (-\infty,\infty)\} = \mathbb{R}$.

$\mathrm{Dom}\,(g \circ f) = \mathrm{Dom}\,(f \circ f) = \mathrm{Dom}\,(g \circ g) = \mathbb{R}$.

$\boxed{8}$ $(f \circ g)(x) = f(x^2) = \sqrt{x^2-3}$, $(g \circ f)(x) = g(\sqrt{x-3}) = x-3$,

$(f \circ f)(x) = f(\sqrt{x-3}) = \sqrt{\sqrt{x-3}-3}$, $(g \circ g)(x) = g(x^2) = (x^2)^2 = x^4$

$\mathrm{Dom}\,(f \circ g) = \{x \mid x \in \mathrm{Dom}\,g \;\&\; g(x) \in \mathrm{Dom}\,f\} = \{x \mid x \in \mathbb{R} \;\&\; x^2 \in [3,\infty)\} = \{x \mid x^2 \geq 3\} = (-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$

$\mathrm{Dom}\,(g \circ f) = \{x \mid x \in \mathrm{Dom}\,f \;\&\; f(x) \in \mathrm{Dom}\,g\} = \{x \mid x \in [3,\infty) \;\&\; \sqrt{x-3} \in \mathbb{R}\} = [3, \infty)$

$\mathrm{Dom}\,(f \circ f) = \{x \mid x \in \mathrm{Dom}\,f \;\&\; f(x) \in \mathrm{Dom}\,f\} = \{x \mid x \in [3,\infty) \;\&\; \sqrt{x-3} \in [3,\infty)\} = [12, \infty)$, since

$\sqrt{x-3} \in [3,\infty) \Rightarrow \sqrt{x-3} \geq 3 \Rightarrow x-3 \geq 9 \Rightarrow x \geq 12$

$\mathrm{Dom}\,(g \circ g) = \mathbb{R}$

$\boxed{9}$ $(f \circ g)(x) = f\left(\frac{x-1}{x+1}\right) = \frac{1}{\frac{x-1}{x+1} - 1} = -\frac{x+1}{2}$, $(g \circ f)(x) = g\left(\frac{1}{x-1}\right) = \frac{\frac{1}{x-1} - 1}{\frac{1}{x-1} + 1} = \frac{2-x}{x}$,

$(f \circ f)(x) = f\left(\frac{1}{x-1}\right) = \frac{1}{\frac{1}{x-1} - 1} = \frac{x-1}{2-x}$, $(g \circ g)(x) = g\left(\frac{x-1}{x+1}\right) = \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1} = -\frac{1}{x}$

$\mathrm{Dom}\,(f \circ g) = \{x \mid x \in \mathrm{Dom}\,g \;\&\; g(x) \in \mathrm{Dom}\,f\} = \{x \mid x \neq -1 \;\&\; \frac{x-1}{x+1} \neq 1\} = \{x \mid x \neq -1\} = (-\infty,-1) \cup (-1,\infty)$

$\mathrm{Dom}\,(g \circ f) = \{x \mid x \in \mathrm{Dom}\,f \;\&\; f(x) \in \mathrm{Dom}\,g\} = \{x \mid x \neq 1 \;\&\; \frac{1}{x-1} \neq -1\} = \{x \mid x \neq 1 \;\&\; x \neq 0\}$

$= (-\infty,0) \cup (0,1) \cup (1,\infty)$, since $\frac{1}{x-1} \neq -1 \Rightarrow 1 \neq -(x-1) \Rightarrow 1 \neq -x+1 \Rightarrow x \neq 0$.

$\mathrm{Dom}\,(f \circ f) = \{x \mid x \in \mathrm{Dom}\,f \;\&\; f(x) \in \mathrm{Dom}\,f\} = \{x \mid x \neq 1 \;\&\; \frac{1}{x-1} \neq 1\} = \{x \mid x \neq 1 \;\&\; x \neq 2\}$

$= (-\infty,1) \cup (1,2) \cup (2,\infty)$

$\mathrm{Dom}\,(g \circ g) = \{x \mid x \in \mathrm{Dom}\,g \;\&\; g(x) \in \mathrm{Dom}\,g\} = \{x \mid x \neq -1 \;\&\; \frac{x-1}{x+1} \neq -1\} = \{x \mid x \neq -1 \;\&\; x \neq 0\}$

$= (-\infty,-1) \cup (-1,0) \cup (0,\infty)$, since $\frac{x-1}{x+1} \neq -1 \Rightarrow x-1 \neq -(x+1) \Rightarrow x-1 \neq -x-1 \Rightarrow$

$2x-1 \neq -1 \Rightarrow 2x \neq 0 \Rightarrow x \neq 0$.

10 $(f \circ g)(x) = f(1-\sqrt{x}) = \sqrt[3]{1-\sqrt{x}}$, $(g \circ f)(x) = g(\sqrt[3]{x}) = 1-\sqrt{\sqrt[3]{x}} = 1-\sqrt[6]{x}$,

$(f \circ f)(x) = f(\sqrt[3]{x}) = \sqrt[3]{\sqrt[3]{x}} = \sqrt[9]{x}$, $(g \circ g)(x) = g(1-\sqrt{x}) = 1-\sqrt{1-\sqrt{x}}$

$\mathrm{Dom}\,(f \circ g) = \{x \mid x \in \mathrm{Dom}\, g \;\&\; g(x) \in \mathrm{Dom}\, f\} = \{x \mid x \in [0,\infty) \;\&\; 1-\sqrt{x} \in \mathbb{R}\} = \{x \mid x \geq 0\}$
$= [0,\infty)$

$\mathrm{Dom}\,(g \circ f) = \{x \mid x \in \mathrm{Dom}\, f \;\&\; f(x) \in \mathrm{Dom}\, g\} = \{x \mid x \in \mathbb{R} \;\&\; \sqrt[3]{x} \in [0,\infty)\} = [0,\infty)$

$\mathrm{Dom}\,(f \circ f) = \mathbb{R}$ (note: the composition of two functions with domain $\mathbb{R}$ has domain $\mathbb{R}$)

$\mathrm{Dom}\,(g \circ g) = \{x \mid x \in \mathrm{Dom}\, g \;\&\; g(x) \in \mathrm{Dom}\, g\} = \{x \mid x \in [0,\infty) \;\&\; 1-\sqrt{x} \in [0,\infty)\} = \{x \mid x \in [0,\infty) \;\&\; x \in [0,1]\}$
$= [0,1]$, where $1-\sqrt{x} \in [0,\infty) \Rightarrow 1-\sqrt{x} \geq 0 \Rightarrow \sqrt{x} \leq 1 \Rightarrow x \in [0,1]$

11 $(f \circ g)(x) = f(\sqrt{2-x}) = \sqrt{(\sqrt{2-x})^2-4} = \sqrt{(2-x)-4} = \sqrt{-2-x}$

$(g \circ f)(x) = g(\sqrt{x^2-4}) = \sqrt{2-\sqrt{x^2-4}}$

$(f \circ f)(x) = f(\sqrt{x^2-4}) = \sqrt{(\sqrt{x^2-4})^2-4} = \sqrt{x^2-4-4} = \sqrt{x^2-8}$

$(g \circ g)(x) = g(\sqrt{2-x}) = \sqrt{2-\sqrt{2-x}}$

$\mathrm{Dom}\,(f \circ g) = \{x \mid x \in \mathrm{Dom}\, g \;\&\; g(x) \in \mathrm{Dom}\, f\} = \{x \mid x \in (-\infty,2] \;\&\; \sqrt{2-x} \in (-\infty,-2] \cup [2,\infty)\}$
$= \{x \mid x \in (-\infty,2] \;\&\; \sqrt{2-x} \in [2,\infty)\}$ (note: $\sqrt{2-x} \in (-\infty,-2]$ is impossible)
$= \{x \mid x \in (-\infty,2] \;\&\; x \in (-\infty,-2]\}$
$= (-\infty,-2]$ (note: $\sqrt{2-x} \in [2,\infty) \Rightarrow \sqrt{2-x} \geq 2 \Rightarrow 2-x \geq 4 \Rightarrow x \leq -2$)

$\mathrm{Dom}\,(g \circ f) = \{x \mid x \in \mathrm{Dom}\, f \;\&\; f(x) \in \mathrm{Dom}\, g\} = \{x \mid x \in (-\infty,-2] \cup [2,\infty) \;\&\; \sqrt{x^2-4} \in (-\infty,2]\}$
$= \{x \mid x \in (-\infty,-2] \cup [2,\infty) \;\&\; x \in [-\sqrt{8},\sqrt{8}]\}$
$= \{x \mid x \in [-\sqrt{8},-2] \cup [2,\sqrt{8}]\} = [-\sqrt{8},-2] \cup [2,\sqrt{8}]$

$\mathrm{Dom}(f \circ f) = \{x \mid x \in (-\infty,-2] \cup [2,\infty) \;\&\; \sqrt{x^2-4} \in (-\infty,-2] \cup [2,\infty)\}$
$= \{x \mid x \in (-\infty,-2] \cup [2,\infty) \;\&\; \sqrt{x^2-4} \in [2,\infty)\}$, where $\sqrt{x^2-4} \geq 2 \Rightarrow x^2-4 \geq 4 \Rightarrow x^2 \geq 8$
$= \{x \mid x \in (-\infty,-2] \cup [2,\infty) \;\&\; x \in (-\infty,-\sqrt{8}) \cup (\sqrt{8},\infty)\} = (-\infty,-\sqrt{8}) \cup (\sqrt{8},\infty)$

$\mathrm{Dom}\,(g \circ g) = \{x \mid x \in (-\infty,2] \cup \sqrt{2-x} \in (-\infty,2]\} = \{x \mid x \in (-\infty,2] \cup x \in [-2,2]\}$
$= [-2,2]$, where $\sqrt{2-x} \in (-\infty,2] \Rightarrow 0 \leq \sqrt{2-x} \leq 2 \Rightarrow 0 \leq 2-x \leq 4 \Rightarrow -2 \leq -x \leq 2$
$\Rightarrow -2 \leq x \leq 2 \Rightarrow x \in [-2,2]$.

12 $(f \circ g)(x) = \dfrac{1}{\sqrt[4]{x^2-4x}}$, $(g \circ f)(x) = \left(\dfrac{1}{\sqrt[4]{x}}\right)^2 - 4\left(\dfrac{1}{\sqrt[4]{x}}\right) = \dfrac{1}{\sqrt{x}} - \dfrac{4}{\sqrt[4]{x}}$,

$(f \circ f)(x) = f\left(1/\sqrt[4]{x}\right) = 1\Big/\sqrt[4]{1/\sqrt[4]{x}} = \dfrac{1}{1/\sqrt[16]{x}} = \sqrt[16]{x}$, $(g \circ g)(x) = x^4 - 8x^3 + 12x^2 + 16x$

$\mathrm{Dom}\,(f \circ g) = \{x \mid x \in \mathbb{R} \;\&\; x^2-4x > 0\} = \{x \mid x(x-4) > 0\} = \{x \mid x < 0 \text{ or } x > 4\}$
$= (-\infty,0) \cup (4,\infty)$

$\mathrm{Dom}\,(g \circ f) = \{x \mid x \in (0,\infty) \;\&\; 1/\sqrt[4]{x} \in \mathbb{R}\} = \{x \mid x \in (0,\infty) \;\&\; x \in (0,\infty)\} = (0,\infty)$

$\mathrm{Dom}\,(f \circ f) = \{x \mid x \in (0,\infty) \;\&\; 1/\sqrt[4]{x} \in (0,\infty)\} = (0,\infty)$

$\mathrm{Dom}\,(g \circ g) = \mathbb{R}$

13 $(f\circ g\circ h)(x) = f(g(h(x))) = f(g(\sqrt[3]{x+3})) = f\left(\sqrt[4]{\sqrt[3]{x+3}-1}\right) = \sqrt{\sqrt[4]{\sqrt[3]{x+3}-1}-2}$

$\mathrm{Dom}(f\circ g\circ h) = \{x \mid x\in \mathrm{Dom}\, h \ \&\ h(x)\in \mathrm{Dom}\, g \ \&\ g(h(x))\in \mathrm{Dom}\, f\}$

$= \{x \mid x\in\mathbb{R} \ \&\ \sqrt[3]{x+3}\in[1,\infty) \ \&\ \sqrt[4]{\sqrt[3]{x+3}-1}\in[2,\infty)\}$

Now, $\sqrt[3]{x+3}\in[1,\infty) \Rightarrow \sqrt[3]{x+3}\ge 1 \Rightarrow x+3\ge 1 \Rightarrow x\ge -2 \Rightarrow x\in[-2,\infty)$

$\sqrt[4]{\sqrt[3]{x+3}-1}\in[2,\infty) \Rightarrow \sqrt[4]{\sqrt[3]{x+3}-1}\ge 2 \Rightarrow \sqrt[3]{x+3}-1\ge 16 \Rightarrow \sqrt[3]{x+3}\ge 17 \Rightarrow$

$x+3\ge 17^3 \Rightarrow x\ge 4910 \Rightarrow x\in[4910,\infty)$

So $\mathrm{Dom}(f\circ g\circ h) = \{x \mid x\in\mathbb{R} \ \&\ x\in[-2,\infty) \ \&\ x\in[4910,\infty)\} = [4910,\infty)$.

14 $(f\circ g\circ h)(x) = f(g(h(x))) = f(g(\sqrt[5]{x})) = f\left(\dfrac{\sqrt[5]{x}}{\sqrt[5]{x}-1}\right) = \sqrt{2\left(\dfrac{\sqrt[5]{x}}{\sqrt[5]{x}-1}\right)}$

$\mathrm{Dom}(f\circ g\circ h) = \left\{x \mid x\in\mathbb{R} \ \&\ \sqrt[5]{x}\in(-\infty,1)\cup(1,\infty) \ \&\ \dfrac{\sqrt[5]{x}}{\sqrt[5]{x}-1}\in[0,\infty)\right\}$

$= \left\{x \mid x\in\mathbb{R} \ \&\ \sqrt[5]{x}\ne 1 \ \&\ \dfrac{\sqrt[5]{x}}{\sqrt[5]{x}-1}\ge 0\right\}$

Now, $\dfrac{\sqrt[5]{x}}{\sqrt[5]{x}-1}\ge 0 \Rightarrow x>1$ <u>or</u> $x\le 0$, so...

$\mathrm{Dom}(f\circ g\circ h) = \{x \mid x\ne 1 \ \&\ x\in(-\infty,0]\cup(1,\infty)\} = (-\infty,0]\cup(1,\infty)$.

15 Let $g(x) = x-8$ & $f(x) = x^4$, so $(f\circ g)(x) = f(g(x)) = f(x-8) = (x-8)^4 = H(x)$

16 Let $g(x) = 5x-3$ & $f(x) = 1/x$, so $(f\circ g)(x) = f(5x-3) = \dfrac{1}{5x-3} = L(x)$

17 Let $h(x) = \sqrt{x}$, $g(x) = x-1$, $f(x) = \sqrt[3]{x}$, so...

$(f\circ g\circ h)(x) = f(g(\sqrt{x})) = f(\sqrt{x}-1) = \sqrt[3]{\sqrt{x}-1} = \Phi(x)$

18 Let $h(x) = 4-\sqrt{x}$, $g(x) = x^2$, $f(x) = \dfrac{9}{x}$, so...

$(f\circ g\circ h)(x) = f(g(4-\sqrt{x})) = f((4-\sqrt{x})^2) = \dfrac{9}{(4-\sqrt{x})^2} = W(x)$