Math 115 Exam #4 Key (Spring 2018)

1 Point estimate for population proportion p is $\hat{p} = x/n = 38/362 \approx 0.1050$. Note that $n\hat{p}, n\hat{q} \geq 5$.

For a 95% confidence interval we have $\alpha = 0.05$, so critical value is $z_{\alpha/2} = z_{0.025} = 1.96$. Margin of error:

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{(0.1050)(1-0.1050)}{362}} = 0.0316,$$

and so the 95% confidence interval for p is

$$(\hat{p}-E, \hat{p}+E) = (0.1050 - 0.0316, 0.1050 + 0.0316) \approx (0.073, 0.137).$$

For a 98% confidence interval we have $\alpha = 0.02$, so critical value is $z_{\alpha/2} = z_{0.01} = 2.33$. Margin of error:

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.33\sqrt{\frac{(0.1050)(1-0.1050)}{362}} = 0.0375,$$

and so the 98% confidence interval for p is

$$(\hat{p}-E, \hat{p}+E) = (0.1050 - 0.0375, 0.1050 + 0.0375) \approx (0.068, 0.143).$$

2 With $\alpha = 0.01$ we have $z_{\alpha/2} = z_{0.005} = 2.575$. The sample proportion of green peas is $\hat{p} = 428/580 = 0.7379$. Margin of error:

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.575\sqrt{\frac{(0.7379)(1-0.7379)}{580}} = 0.0470.$$

The 99% confidence interval for p:

$$(\hat{p}-E, \hat{p}+E) = (0.7379 - 0.0470, 0.7379 + 0.0470) \approx (0.691, 0.785).$$

The 99% confidence interval for the percentage of green peas $100p\%$:

$$(69.1\%, 78.5\%).$$

Since this interval contains 75% the results do not contradict the theory.

3a Assuming $\hat{p} = \hat{q} = 0.5$, with $\alpha = 0.005$ we have

$$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2 = (0.5)(0.5)\left(\frac{2.575}{0.04}\right)^2 \approx 1036.04,$$

and so the sample size should be at least 1037.

3b Now we have $\hat{p} = 0.26$, so that

$$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2 = (0.26)(0.74)\left(\frac{2.575}{0.04}\right)^2 \approx 797.33.$$

The sample size should be at least 798.

4 The sample mean and sample standard deviation are $\overline{x} = 98.9$ and $s = 42.3$. Degrees of freedom is $n - 1 = 18$. Margin of error with $\alpha = 0.02$:

$$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = 2.552 \cdot \frac{42.3}{\sqrt{19}} = 24.77.$$

The 98% confidence interval for mean wake time μ is

$$(\overline{x} - E, \overline{x} + E) \approx (74.1, 123.7).$$

Bigpharmazone does not appear to be effective at the $\alpha = 0.02$ level since the mean wake time of 102.8 minutes before treatment lies within the 98% confidence interval.

5 From the data we calculate $\overline{x} = 6.53$ and $s = 2.339$. For a 90% confidence interval we have $\alpha = 0.10$, and with the degrees of freedom being $n - 1 = 9$ we find $t_{\alpha/2} = t_{0.05} = 1.833$. The margin of error is thus

$$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = 1.833 \cdot \frac{2.339}{\sqrt{10}} = 1.356.$$

The 90% confidence interval for the mean arsenic amount μ is

$$(\overline{x} - E, \overline{x} + E) \approx (5.17, 7.89).$$

This confidence interval cannot be used to describe arsenic levels in places as faraway as Arkansas where the soil is different.

6 Here d.f. $= n - 1 = 18$ and $\alpha = 0.02$, and so from the chi-square distribution table we have $\chi^2_R = 34.805$ and $\chi^2_L = 7.015$. The 98% confidence interval for the population standard deviation σ is

$$\left(\sqrt{\frac{(n-1)s^2}{\chi^2_R}}, \sqrt{\frac{(n-1)s^2}{\chi^2_L}}\right) = \left(\sqrt{\frac{18(42.3)^2}{34.805}}, \sqrt{\frac{18(42.3)^2}{7.015}}\right) = (30.4, 67.8).$$

7a Letting p be the proportion of lawsuits dropped or dismissed, we have

$$\begin{cases} H_0 : p \le 0.5 \\ H_a : p > 0.5 \ \text{(claim)}. \end{cases}$$

7b Critical value is $z_\alpha = z_{0.01} = 2.33$.

7c Rejection region: $(2.33, \infty)$.

7d With $\hat{p} = 706/1228 = 0.575$, the standardized test statistic value is

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.575 - 0.5}{\sqrt{(0.5)(0.5)/1228}} = 5.26.$$

7e Since $z = 5.26$ lies in the rejection region, we reject H_0. Therefore the claim cannot be rejected.

8a We have

$$\begin{cases} H_0 : \mu \leq 0 \\ H_a : \mu > 0 \ \text{ (claim).} \end{cases}$$

8b Critical value with degrees of freedom $n - 1 = 48$ is $t_\alpha = t_{0.05} = 1.676$.

8c Rejection region: $(1.676, \infty)$.

8d Standardized test statistic value:

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}} = \frac{0.4 - 0}{21.0/\sqrt{49}} = 0.13.$$

8e Since $t = 0.13$ is not in the rejection region, we fail to reject H_0. The claim can be rejected.