Math 115 Exam #3 Key (Spring 2018)

1a The mean μ is:

$$\sum x\mathsf{P}(x) = 0(0.358) + 1(0.439) + 2(0.179) + 3(0.024) = 0.869.$$

1b The standard deviation σ is:

$$\begin{aligned}&\sqrt{\sum (x-\mu)^2\,\mathsf{P}(x)} = \sqrt{\sum (x-.869)^2\,\mathsf{P}(x)}\\ &= \sqrt{(0-.869)^2(.358) + (1-.869)^2(.439) + (2-.869)^2(.179) + (3-.869)^2(.024)}\\ &= \sqrt{0.615839} \approx 0.785.\end{aligned}$$

1c Probability is $\mathsf{P}(1) + \mathsf{P}(2) + \mathsf{P}(3) = 0.439 + 0.179 + 0.024 = 0.642$.

2a Letting X denote the number of questions answered correctly,

$$\mathsf{P}(X = 4) = {}_8C_4(0.25)^4(0.75)^4 = 0.0865.$$

2b A score of at least 70% results from getting at least 6 questions right, so probability is

$$\begin{aligned}\mathsf{P}(X \geq 6) &= \mathsf{P}(X = 6) + \mathsf{P}(X = 7) + \mathsf{P}(X = 8)\\ &= {}_8C_6(0.25)^6(0.75)^2 + {}_8C_7(0.25)^7(0.75)^1 + {}_8C_8(0.25)^8(0.75)^0\\ &= 0.003845 + 0.0003662 + 0.00001526\\ &\approx 0.00423.\end{aligned}$$

2c $\mathsf{P}(X \geq 1) = 1 - \mathsf{P}(X < 1) = 1 - \mathsf{P}(X = 0) = 1 - (0.75)^8 = 0.900$.

3a This is a Poisson experiment with parameter $\lambda = 0.6$, and so if the random variable X counts the number of hurricanes that strike the U.S. in a year, then

$$\mathsf{P}(X = k) = \frac{\lambda^k e^{-\lambda}}{k!} = \frac{0.6^k e^{-0.6}}{k!}$$

for $k = 0, 1, 2, \ldots$. Thus the probability exactly two hurricanes strike is

$$\mathsf{P}(X = 2) = \frac{0.6^2 e^{-0.6}}{2!} \approx 0.0988.$$

3b We have

$$\begin{aligned}\mathsf{P}(X > 2) &= 1 - \mathsf{P}(X \leq 2) = 1 - \mathsf{P}(X = 0) - \mathsf{P}(X = 1) - \mathsf{P}(X = 2)\\ &= 1 - \frac{0.6^0 e^{-0.6}}{0!} - \frac{0.6^1 e^{-0.6}}{1!} - \frac{0.6^2 e^{-0.6}}{2!}\\ &= 1 - 0.5488 - 0.3293 - 0.0988 = 0.0231.\end{aligned}$$

4 Area $= \mathsf{P}(-1.2 \leq Z \leq 2.1) = \mathsf{P}(Z \leq 2.1) - \mathsf{P}(Z < -1.2) = 0.9821 - 0.1151 = 0.8670.$

5 $\mathsf{P}(Z < -2.58 \text{ or } Z > 2.58) = 2\mathsf{P}(Z < -2.58) = 2(0.0049) = 0.0098.$

6 Letting random variable X be back-to-knee lengths of men, we have $\mathsf{P}(X \leq P_{80}) = 0.80$. Standardizing,

$$\mathsf{P}\left(Z \leq \frac{P_{80} - 23.5}{1.1}\right) = 0.80,$$

and from the normal distribution table we find that

$$\frac{P_{80} - 23.5}{1.1} = 0.84 \quad \Rightarrow \quad P_{80} = 0.84(1.1) + 23.5 = 24.424 \approx 24.42.$$

7a Let X be the random variable that gives the resistance of a wire. We must find $\mathsf{P}(X > 0.142)$. Using the standard normal distribution table,

$$\begin{aligned} \mathsf{P}(X > 0.142) &= \mathsf{P}\left(\frac{X - 0.13}{0.005} > \frac{0.142 - 0.13}{0.005}\right) = \mathsf{P}(Z > 2.40) \\ &= 1 - \mathsf{P}(Z \leq 2.40) = 1 - 0.9918 = 0.0082. \end{aligned}$$

7b $\mathsf{P}(X < 0.128) = \mathsf{P}\left(Z < \dfrac{0.128 - 0.13}{0.005} = -0.4\right) = 0.3446.$

7c We have

$$\begin{aligned} \mathsf{P}(0.123 < X < 0.139) &= \mathsf{P}\left(\frac{0.123 - 0.13}{0.005} < \frac{X - 0.13}{0.005} < \frac{0.139 - 0.13}{0.005}\right) \\ &= \mathsf{P}(-1.40 < Z < 1.80) \\ &= \mathsf{P}(Z < 1.80) - \mathsf{P}(Z \leq -1.40) \\ &= 0.9641 - 0.0808 = 0.8833. \end{aligned}$$

8 Let the random variable $\overline{X}$ denote the mean price, so by the Central Limit Theorem $\overline{X}$ is approximately normally distributed with mean $\mu_{\overline{X}} = 4.220$ and standard deviation

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.082}{\sqrt{36}} = 0.0137.$$

Now,

$$\begin{aligned} \mathsf{P}(\overline{X} > 4.28) &= 1 - \mathsf{P}(\overline{X} \leq 4.28) = 1 - \mathsf{P}\left(Z \leq \frac{4.28 - \mu_{\overline{X}}}{\sigma_{\overline{X}}}\right) \\ &= 1 - \mathsf{P}(Z \leq 4.38) = 1 - 0.9999 = 0.0001. \end{aligned}$$