Math 115 Exam #2 Key (Spring 2018)

1 Median $= \frac{28+29}{2} = 28.5$, Mode $= 29$,

$$\text{Mean} = \frac{1}{14}\sum_{k=1}^{14} x_k = \frac{384}{14} = \frac{192}{7} \approx 27.429,$$

and Midrange $= \frac{18+36}{2} = 27$.

2 First, Range $= 36 - 18 = 18$. The sample standard deviation is

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} = \sqrt{\frac{\sum(x-192/7)^2}{13}} \approx 4.5525 \approx 4.55,$$

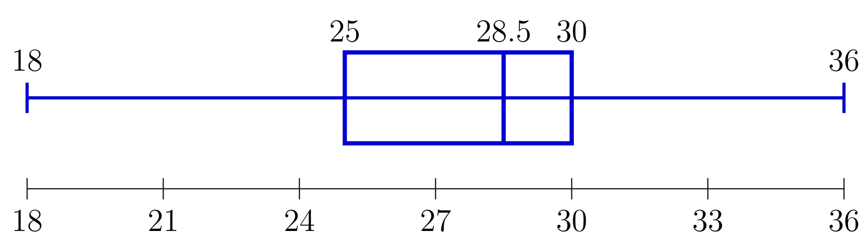
and the sample variance is $s^2 = (4.5525)^2 \approx 20.73$. Finally, the coefficient of variation is

$$\text{CV} = \frac{s}{\bar{x}} \cdot 100 = \frac{4.5525}{192/7} \cdot 100 \approx 16.60.$$

3a The Median is 28.5, and so $Q_2 = 28.5$. Now we divide the data set into two halves: its smallest 7 values and its largest 7 values. The median of the smallest 7 values is $Q_1 = 25$, and the median of the largest 7 values is $Q_3 = 30$. Thus:

$$Q_1 = 25, \quad Q_2 = 28.5, \quad Q_3 = 30.$$

3b



4 The number of values less than 20 is 1, so

$$\text{Percentile of } 20 = \frac{\text{Number of values less than 20}}{\text{Total number of values}} \cdot 100 = \frac{1}{14} \cdot 100 \approx 7.14 \to 7.$$

That is, the value 20 is in the 7th percentile.

The number of values less than 29 is 7, so

$$\text{Percentile of } 29 = \frac{\text{Number of values less than 29}}{\text{Total number of values}} \cdot 100 = \frac{7}{14} \cdot 100 = 50.$$

That is, the value 29 is in the 50th percentile.

5 Take the weighted average:

$$.24(63\%) + .20(89\%) + .18(81\%) + .12(100\%) + .14(53\%) + .10(39\%) + .02(50\%) = 71.82\%.$$

6 Probability is $\frac{1}{4} \cdot \frac{12}{52} = \frac{12}{208} \approx 0.0577$.

7a $\frac{414}{2152} \approx 0.192.$

7b $\frac{1161}{2152} + \frac{46}{2152} = \frac{1207}{2152} \approx 0.561.$

7c $\frac{659}{1228} \approx 0.537.$

7d $\frac{323}{531} \approx 0.608.$

8 Let E_1 be the event of a puncture, and E_2 the event of a smashed corner. By the Addition Rule,

$$\mathsf{P}(E_1 \text{ or } E_2) = \mathsf{P}(E_1) + \mathsf{P}(E_2) - \mathsf{P}(E_1 \text{ and } E_2) = 0.07 + 0.04 - 0.005 = 0.105.$$

9 ${}_{18}P_5 = \dfrac{18!}{(18-5)!} = \dfrac{18!}{13!} = (14)(15)(16)(17)(18) = 1{,}028{,}160.$

10 Number of customizations is $3 \cdot 2 \cdot 6 \cdot 5 \cdot 9 = 1620.$

11 There are $n = 11$ letters, with number of A's being $n_A = 4$ and number of L's being $n_L = 3$. All other letters occur once. Number of ways the letters can be arranged:

$$\frac{n!}{n_A!\, n_L!} = \frac{11!}{4!\, 3!} = 277{,}200.$$