Math 115 Exam #5 Key (Spring 2017)

1a We have

$$\begin{cases} H_0 : \sigma = 118.4 \ \text{(claim)} \\ H_a : \sigma \neq 118.4. \end{cases}$$

1b Using the χ^2 distribution table with 19 degrees of freedom, we find the critical values are $\chi^2_L = 6.844$ and $\chi^2_R = 38.582$.

1c Rejection region is $R = (-\infty, 6.844) \cup (38.582, \infty)$.

1d Test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{19(85.6)^2}{(118.4)^2} = 9.93.$$

1e Since $9.93 \notin R$, we cannot reject H_0. Since H_0 is the claim, the claim is supported at the $\alpha = 0.01$ significance level.

2a Let μ_A and μ_B be the mean wind speed in Region A and Region B, respectively.

$$\begin{cases} H_0 : \mu_A \geq \mu_B \\ H_a : \mu_A < \mu_B \ \text{(claim)}. \end{cases}$$

2b The populations are normally distributed, the samples are large, and the population standard deviations are known. Thus we may use the standard normal distribution table (z table) to find the critical value: $z_0 = -1.645$.

2c Rejection region is $R = (-\infty, -1.645)$.

2d Test statistic is

$$z = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \frac{(21 - 22.7) - 0}{\sqrt{\dfrac{3.4^2}{60} + \dfrac{4.9^2}{60}}} \approx -2.21.$$

2e Since $-2.21 \in R$, we reject H_0. Since H_a is the claim, the claim is supported at the $\alpha = 0.05$ significance level.

3a Let μ_1 be the mean annual cost of food for dogs, and μ_2 the cost for cats. Based on the data the claim would be that $\mu_1 > \mu_2$.

$$\begin{cases} H_0 : \mu_1 \leq \mu_2 \\ H_a : \mu_1 > \mu_2 \ \text{(claim)}. \end{cases}$$

3b The samples are small (less than 30), and so we use the t distribution table with degrees of freedom equalling the smaller of $n_1 - 1 = 15$ and $n_2 - 1 = 17$ (i.e. 15). Critical value is $t_0 = 1.341$.

3c Rejection region is $R = (1.341, \infty)$.

3d Standardized test statistic value:

$$z = \frac{\overline{x} - \mu}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{239 - 203}{\sqrt{\frac{32^2}{16} + \frac{21^2}{18}}} \approx 3.83.$$

3e Since $3.83 \in R$, we reject H_0. Since H_a is the claim, the claim is supported at the $\alpha = 0.10$ significance level.