Math 115 Exam #4 Key (Spring 2017)

1a Point estimate for population proportion p is $\hat{p} = x/n = 153/5924 \approx 0.0258$. Note that $n\hat{p}, n\hat{q} \geq 5$. Critical value: $z_c = 1.645$. Margin of error:

$$E = z_c\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.645\sqrt{\frac{(0.0258)(1-0.0258)}{5924}} = 0.0034.$$

90% confidence interval for p:

$$(\hat{p} - E, \hat{p} + E) = (0.0258 - 0.0034, 0.0258 + 0.0034) = (0.0224, 0.0292).$$

1b Only the critical value changes: $z_c = 2.575$. Margin of error:

$$E = z_c\sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.575\sqrt{\frac{(0.0258)(1-0.0258)}{5924}} = 0.0053.$$

99% confidence interval for p:

$$(\hat{p} - E, \hat{p} + E) = (0.0258 - 0.0053, 0.0258 + 0.0053) = (0.0205, 0.0311).$$

2a Assuming $\hat{p} = \hat{q} = 0.5$, we have

$$n = \hat{p}\hat{q}\left(\frac{z_c}{E}\right)^2 = (0.5)(0.5)\left(\frac{1.96}{0.03}\right)^2 \approx 1067.1,$$

and so the sample size should be at least 1068.

2b Now we have $\hat{p} = 0.83$, so that

$$n = \hat{p}\hat{q}\left(\frac{z_c}{E}\right)^2 = (0.83)(0.17)\left(\frac{1.96}{0.03}\right)^2 \approx 602.3.$$

The sample size should be at least 603.

3 The sample mean and sample variance are

$$\overline{x} = \frac{1}{n}\sum x = 61.5 \quad \text{and} \quad s^2 = \frac{\sum(x-\overline{x})^2}{n-1} = 25.54.$$

Here d.f. $= n - 1 = 11$, and so from the chi-square distribution table we have $\chi_R^2 = 24.725$ and $\chi_L^2 = 3.053$. The 98% confidence interval for variance σ^2 is

$$\left(\frac{(n-1)s^2}{\chi_R^2}, \frac{(n-1)s^2}{\chi_L^2}\right) = \left(\frac{11(25.54)}{24.725}, \frac{11(25.54)}{3.053}\right) = (11.36, 92.02).$$

The 98% confidence interval for standard deviation σ is

$$\left(\sqrt{11.36}, \sqrt{92.02}\right) = (3.37, 9.59).$$

4a We have
$$\begin{cases} H_0 : \mu \leq 40 \ \ \text{(claim)} \\ H_a : \mu > 40. \end{cases}$$

4b Critical value: 2.33.

4c Rejection region: $(2.33, \infty)$.

4d Standardized test statistic value:
$$Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{41.1 - 40}{7.5/\sqrt{20}} = 0.656.$$

4e Since $Z = 0.656$ is not in the rejection region, we fail to reject H_0. Therefore the company's claim cannot be rejected.

5a We have
$$\begin{cases} H_0 : \mu = 30 \ \ \text{(claim)} \\ H_a : \mu \neq 30. \end{cases}$$

5b Critical values: 2.898 and -2.898.

5c Using the t-distribution table with d.f. $= 17$, the rejection region is:
$$(-\infty, -2.898) \cup (2.898, \infty).$$

5d Standardized test statistic value:
$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{28.5 - 30}{1.7/\sqrt{18}} = -3.74.$$

5e Since $t = -3.74$ is in the rejection region, we reject H_0. Therefore the manufacturer's claim can be rejected.

6a We have
$$\begin{cases} H_0 : p \leq 0.25 \\ H_a : p > 0.25 \ \ \text{(claim)} \end{cases}$$

6b Critical value: 2.33.

6c Rejection region: $(2.33, \infty)$.

6d With $\hat{p} = 152/428 \approx 0.36$, the standardized test statistic value is
$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.36 - 0.25}{\sqrt{(0.25)(0.75)/428}} = 5.26.$$

6e Since $z = 5.26$ is in the rejection region, we reject H_0. Therefore Mendel's claim can not be rejected.