Math 115 Exam #3 Key (Spring 2017)

1 Area $= \mathsf{P}(-1.2 \le Z \le 2.1) = \mathsf{P}(Z \le 2.1) - \mathsf{P}(Z < -1.2) = 0.9821 - 0.1151 = 0.8670.$

2 $\mathsf{P}(Z < -2.58 \text{ or } Z > 2.58) = 2\mathsf{P}(Z < -2.58) = 2(0.0049) = 0.0098.$

3a Let X be the random variable that gives the resistance of a wire. We must find $\mathsf{P}(X > 0.142)$. Using the standard normal distribution table,

$$\begin{aligned}\mathsf{P}(X > 0.142) &= \mathsf{P}\left(\frac{X - 0.13}{0.005} > \frac{0.142 - 0.13}{0.005}\right) = \mathsf{P}(Z > 2.40)\\ &= 1 - \mathsf{P}(Z \le 2.40) = 1 - 0.9918 = 0.0082.\end{aligned}$$

3b $\mathsf{P}(X < 0.128) = \mathsf{P}\left(Z < \dfrac{0.128 - 0.13}{0.005} = -0.4\right) = 0.3446.$

3c We have

$$\begin{aligned}\mathsf{P}(0.123 < X < 0.139) &= \mathsf{P}\left(\frac{0.123 - 0.13}{0.005} < \frac{X - 0.13}{0.005} < \frac{0.139 - 0.13}{0.005}\right)\\ &= \mathsf{P}(-1.40 < Z < 1.80)\\ &= \mathsf{P}(Z < 1.80) - \mathsf{P}(Z \le -1.40)\\ &= 0.9641 - 0.0808 = 0.8833.\end{aligned}$$

4 We find z such that $\mathsf{P}(Z \ge z) = 0.785$, or equivalently

$$\mathsf{P}(Z < z) = 1 - \mathsf{P}(Z \ge z) = 1 - 0.785 = 0.215.$$

The closest value in the table to 0.215 is 0.2148, which gives $z = -0.79$.

5a The z-score is such that $\mathsf{P}(Z \ge z) = 0.1$, or equivalently $\mathsf{P}(Z < z) = 0.9$. The closest value in the table is $z = 1.28$. Now,

$$z = 1.28 \quad\Rightarrow\quad \frac{x - \mu}{\sigma} = 1.28 \quad\Rightarrow\quad x = 1.28\sigma + \mu = 1.28(3) + 10.4 = 14.24,$$

and so the smallest consumption in the top 10% is 14.2 pounds.

5b $\mathsf{P}(Z \le z) = 0.05$ implies $z = -1.645$, so

$$x = -1.645(3) + 10.4 = 5.465,$$

and the largest consumption in the bottom 5% is 5.5 pounds.

6 Let the random variable $\overline{X}$ denote the mean price, so by the Central Limit Theorem $\overline{X}$ is approximately normally distributed with mean $\mu_{\overline{X}} = 4.117$ and standard deviation

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.049}{\sqrt{38}} = 0.0079.$$

Now,

$$\begin{aligned}\mathsf{P}(4.128 \le \overline{X} \le 4.143) &= \mathsf{P}\left(\frac{4.128 - \mu_{\overline{X}}}{\sigma_{\overline{X}}} \le Z \le \frac{4.143 - \mu_{\overline{X}}}{\sigma_{\overline{X}}}\right)\\ &= \mathsf{P}(1.39 \le Z \le 3.29) = \mathsf{P}(Z \le 3.29) - \mathsf{P}(Z < 1.39)\\ &= 0.9995 - 0.9177 = 0.0818.\end{aligned}$$

7a Let P_{bin} and P_{nor} denote probabilities calculated using the binomial and normal distributions, respectively. We have a binomial distribution with $n = 500$, $p = 0.35$, and $q = 0.65$, and it is approximated with a normal distribution with $\mu = np = 175$ and $\sigma = \sqrt{npq} = 10.665$. Now,

$$\begin{aligned}\mathsf{P}_{\text{bin}}(X = 175) &\approx \mathsf{P}_{\text{nor}}(174.5 \le X \le 175.5) = \mathsf{P}_{\text{nor}}\left(\frac{174.5 - \mu}{\sigma} \le Z \le \frac{175.5 - \mu}{\sigma}\right)\\ &= \mathsf{P}_{\text{nor}}(-0.047 \le Z \le 0.047) = \mathsf{P}_{\text{nor}}(Z \le 0.05) - \mathsf{P}_{\text{nor}}(Z \le -0.05)\\ &= 0.5299 - 0.4801 = 0.0498.\end{aligned}$$

7b We have

$$\mathsf{P}_{\text{bin}}(X \le 225) \approx \mathsf{P}_{\text{nor}}(X \le 225.5) = \mathsf{P}_{\text{nor}}\left(Z \le \frac{225.5 - 175}{10.665}\right) = \mathsf{P}_{\text{nor}}(Z \le 4.74) = 1.$$

8 For the 90% confidence interval we have $z_c = z_{0.90} = 1.645$, so margin of error is

$$E = z_c \frac{\sigma}{\sqrt{n}} = 1.645\left(\frac{6.76}{\sqrt{42}}\right) \approx 1.72,$$

and the confidence interval is

$$(\overline{x} - E, \overline{x} + E) = (22.4 - 1.72, 22.4 + 1.72) \approx (20.68, 24.12).$$

For the 95% confidence interval we have $z_c = z_{0.95} = 1.96$, so margin of error is

$$E = z_c \frac{\sigma}{\sqrt{n}} = 1.96\left(\frac{6.76}{\sqrt{42}}\right) \approx 2.04,$$

and the confidence interval is

$$(\overline{x} - E, \overline{x} + E) = (22.4 - 2.04, 22.4 + 2.04) \approx (20.36, 24.44).$$

9 Here $n = 7$, $\overline{x} = 173$, $s = 29.60$, $c = 0.98$, and d.f. $= 6$. From the t-table we find that $t_{0.98} = 3.143$, and so the confidence interval is

$$\left(\overline{x} - t_c \frac{s}{\sqrt{n}}, \overline{x} + t_c \frac{s}{\sqrt{n}}\right) = (137.8, 208.2).$$