Math 115 Exam #4 Key (Fall 2016)

1a Point estimate for population proportion p is $\hat{p} = x/n = 734/2303 \approx 0.319$. Note that $n\hat{p}, n\hat{q} \geq 5$. Critical value: $z_{0.95} = 1.96$. Margin of error:

$$E = z_{0.95}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{(0.319)(1-0.319)}{2303}} = 0.019.$$

95% confidence interval for p:

$$(\hat{p} - E, \hat{p} + E) = (0.319 - 0.019, 0.319 + 0.019) = (0.300, 0.338).$$

1b Point estimate for population proportion p is $\hat{p} \approx 0.319$. Critical value: $z_{0.98} = 2.33$. Margin of error:

$$E = z_{0.98}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.33\sqrt{\frac{(0.319)(1-0.319)}{2303}} = 0.023.$$

98% confidence interval for p:

$$(\hat{p} - E, \hat{p} + E) = (0.319 - 0.023, 0.319 + 0.023) = (0.296, 0.342).$$

2a Assuming $\hat{p} = \hat{q} = 0.5$, we have

$$n = \hat{p}\hat{q}\left(\frac{z_{0.99}}{E}\right)^2 = (0.5)(0.5)\left(\frac{2.575}{0.02}\right)^2 \approx 4144.1,$$

and so the sample size should be at least 4145.

2b Now we have $\hat{p} = 0.87$, so that

$$n = \hat{p}\hat{q}\left(\frac{z_{0.99}}{E}\right)^2 = (0.87)(0.13)\left(\frac{2.575}{0.02}\right)^2 \approx 1874.8.$$

The sample size should be at least 1875.

3 The sample mean is $\overline{X} = 29.43/18 = 1.635$, and the sample variance is

$$s^2 = \frac{\sum(X - \overline{X})^2}{n-1} = \frac{1.09045}{17} = 0.06414.$$

Here d.f. $= n - 1 = 17$, and so from the chi-square distribution table we have $\chi^2_R = 27.587$ and $\chi^2_L = 8.672$. The 90% confidence interval for variance σ^2 is

$$\left(\frac{(n-1)s^2}{\chi^2_R}, \frac{(n-1)s^2}{\chi^2_L}\right) = \left(\frac{17(0.06414)}{27.587}, \frac{17(0.06414)}{8.672}\right) = (0.0395, 0.1257).$$

The 90% confidence interval for standard deviation σ is

$$\left(\sqrt{0.0395}, \sqrt{0.1257}\right) = (0.1987, 0.3545).$$

4a We have

$$\begin{cases} H_0 : \mu \leq 40 \ \ \text{(claim)} \\ H_a : \mu > 40. \end{cases}$$

4b Critical value: 2.33.

4c Rejection region: $(2.33, \infty)$.

4d Standardized test statistic value:

$$Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{41.1 - 40}{7.5/\sqrt{20}} = 0.656.$$

4e Since $Z = 0.656$ is not in the rejection region, we fail to reject H_0. Therefore the company's claim cannot be rejected.

5a We have

$$\begin{cases} H_0 : \mu = 30 \ \ \text{(claim)} \\ H_a : \mu \neq 30. \end{cases}$$

5b Critical values: 2.898 and -2.898.

5c Using the t-distribution table with d.f. = 17, the rejection region is:

$$(-\infty, -2.898) \cup (2.898, \infty).$$

5d Standardized test statistic value:

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{28.5 - 30}{1.7/\sqrt{18}} = -3.74.$$

5e Since $t = -3.74$ is in the rejection region, we reject H_0. Therefore the manufacturer's claim can be rejected.

6a We have

$$\begin{cases} H_0 : p \geq 0.46 \ \ \text{(claim)} \\ H_a : p < 0.46. \end{cases}$$

6b Critical value: -1.645.

6c Rejection region: $(-\infty, -1.645)$.

6d Standardized test statistic value:

$$Z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.41 - 0.46}{\sqrt{(0.46)(0.54)/300}} = -1.74.$$

6e Since $Z = -1.74$ is in the rejection region, we reject H_0. Therefore the research center's claim can be rejected.