Math 115 Exam #3 Key (Fall 2016)

1 Area $= \mathsf{P}(Z \geq -1.38) = 1 - \mathsf{P}(Z < -1.38) = 1 - 0.0838 = 0.9162$.

2 Probability is

$$\begin{aligned}\mathsf{P}(0 < Z < 0.525) &= \mathsf{P}(Z < 0.525) - \mathsf{P}(Z < 0)\\ &= \frac{\mathsf{P}(Z < 0.53) + \mathsf{P}(Z < 0.52)}{2} - \mathsf{P}(Z < 0)\\ &= \frac{0.7019 + 0.6985}{2} - 0.5 = 0.2002.\end{aligned}$$

3a Let X be the random variable that gives the alkalinity level of a water specimen from the river. We must find $\mathsf{P}(X > 45)$. Using the standard normal distribution table,

$$\begin{aligned}\mathsf{P}(X > 45) &= \mathsf{P}\left(\frac{X - 50}{3.2} > \frac{45 - 50}{3.2}\right) = \mathsf{P}(Z > -1.5625) \approx \mathsf{P}(Z > -1.56)\\ &= 1 - \mathsf{P}(Z \leq -1.56) = 1 - 0.0594 = 0.9406.\end{aligned}$$

3b We have

$$\mathsf{P}(X \leq 55) = \mathsf{P}\left(Z \leq \frac{55 - 50}{3.2} = 1.5625\right) \approx \mathsf{P}(Z \leq 1.56) = 0.9406,$$

the same answer as in part (a). This should not be surprising considering the symmetry of the normal distribution about μ in general.

3c We have

$$\begin{aligned}\mathsf{P}(51 < X < 52) &= \mathsf{P}\left(\frac{51 - 50}{3.2} < \frac{X - 50}{3.2} < \frac{52 - 50}{3.2}\right)\\ &= \mathsf{P}(0.3125 < Z < 0.6250)\\ &= \mathsf{P}(Z \leq 0.6250) - \mathsf{P}(Z < 0.3125)\\ &\approx \frac{\mathsf{P}(Z \leq 0.62) + \mathsf{P}(Z \leq 0.63)}{2} - \mathsf{P}(Z \leq 0.31)\\ &= \frac{0.7324 + 0.7357}{2} - 0.6217 \approx 0.1124.\end{aligned}$$

Note that we round the final probability value to four decimal places in accord with the table and significant figure conventions.

4 We find z such that $\mathsf{P}(Z \geq z) = 0.785$, or equivalently

$$\mathsf{P}(Z < z) = 1 - \mathsf{P}(Z \geq z) = 1 - 0.785 = 0.215.$$

The closest value in the table to 0.215 is 0.2148, which gives $z = -0.79$.

5a The z-score is such that $\mathsf{P}(Z \geq z) = 0.1$, or equivalently $\mathsf{P}(Z < z) = 0.9$. The closest value in the table is $z = 1.28$. Now,

$$z = 1.28 \quad \Rightarrow \quad \frac{x - \mu}{\sigma} = 1.28 \quad \Rightarrow \quad x = 1.28\sigma + \mu = 1.28(3) + 10.4 = 14.24,$$

and so the smallest consumption in the top 10% is 14.2 pounds.

5b $\mathsf{P}(Z \leq z) = 0.05$ implies $z = -1.645$, so

$$x = -1.645(3) + 10.4 = 5.465,$$

and the largest consumption in the bottom 5% is 5.5 pounds.

6 Let the random variable $\overline{X}$ denote the mean price, so by the Central Limit Theorem $\overline{X}$ is approximately normally distributed with mean $\mu_{\overline{X}} = 4.117$ and standard deviation

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.049}{\sqrt{38}} = 0.0079.$$

Now,

$$\begin{aligned} \mathsf{P}(4.128 \leq \overline{X} \leq 4.143) &= \mathsf{P}\left(\frac{4.128 - \mu_{\overline{X}}}{\sigma_{\overline{X}}} \leq Z \leq \frac{4.143 - \mu_{\overline{X}}}{\sigma_{\overline{X}}}\right) \\ &= \mathsf{P}(1.39 \leq Z \leq 3.29) = \mathsf{P}(Z \leq 3.29) - \mathsf{P}(Z < 1.39) \\ &= 0.9995 - 0.9177 = 0.0818. \end{aligned}$$

7a Let P_{bin} and P_{nor} denote probabilities calculated using the binomial and normal distributions, respectively. We have a binomial distribution with $n = 500$, $p = 0.35$, and $q = 0.65$, and it is approximated with a normal distribution with $\mu = np = 175$ and $\sigma = \sqrt{npq} = 10.665$. Now,

$$\begin{aligned} \mathsf{P}_{\text{bin}}(X = 175) &\approx \mathsf{P}_{\text{nor}}(174.5 \leq X \leq 175.5) = \mathsf{P}_{\text{nor}}\left(\frac{174.5 - \mu}{\sigma} \leq Z \leq \frac{175.5 - \mu}{\sigma}\right) \\ &= \mathsf{P}_{\text{nor}}(-0.047 \leq Z \leq 0.047) = \mathsf{P}_{\text{nor}}(Z \leq 0.05) - \mathsf{P}_{\text{nor}}(Z \leq -0.05) \\ &= 0.5299 - 0.4801 = 0.0498. \end{aligned}$$

7b We have

$$\mathsf{P}_{\text{bin}}(X \leq 225) \approx \mathsf{P}_{\text{nor}}(X \leq 225.5) = \mathsf{P}_{\text{nor}}\left(Z \leq \frac{225.5 - 175}{10.665}\right) = \mathsf{P}_{\text{nor}}(Z \leq 4.74) = 1.$$

8 For the 90% confidence interval we have $z_c = z_{0.90} = 1.645$, so margin of error is

$$E = z_c \frac{\sigma}{\sqrt{n}} = 1.645\left(\frac{6.7}{\sqrt{36}}\right) \approx 1.84,$$

and the confidence interval is

$$(\overline{x} - E, \overline{x} + E) = (23 - 1.84, 23 + 1.84) \approx (21.2, 24.8).$$

For the 95% confidence interval we have $z_c = z_{0.95} = 1.96$, so margin of error is

$$E = z_c \frac{\sigma}{\sqrt{n}} = 1.96\left(\frac{6.7}{\sqrt{36}}\right) \approx 2.19,$$

and the confidence interval is

$$(\overline{x} - E, \overline{x} + E) = (23 - 2.19, 23 + 2.19) \approx (20.8, 25.2).$$

9 Here $n = 9$, $\overline{x} = 173$, $s = 29.60$, $c = 0.98$, and d.f. $= 8$. From the t-table we find that $t_{0.98} = 2.896$, and so the confidence interval is

$$\left(\overline{x} - t_c \frac{s}{\sqrt{n}}, \overline{x} + t_c \frac{s}{\sqrt{n}}\right) = (144.4, 201.6).$$