Math 115 Exam #2 Key (Fall 2016)

1 Letting H denote heads and T denote tails:

$$\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}.$$

2 By the Fundamental Counting Principle: $(3)(3)(4)(6)(2) = 432$ ways.

3 Probability of diamond is $\frac{13}{52} = \frac{1}{4}$, and probability of king is $\frac{4}{52} = \frac{1}{13}$. By the Multiplication Rule the probability of a diamond and then a king is $\frac{1}{4} \cdot \frac{1}{13} = \frac{1}{52}$.

4a $\frac{46}{2152} \approx 0.0214$.

4b $\frac{531}{2152} + \frac{414}{2152} = \frac{945}{2152} \approx 0.439$.

4c $\frac{659}{1228} \approx 0.537$.

4d $\frac{659}{1161} \approx 0.568$.

5 Let E_1 be the event of a puncture, and E_2 the event of a smashed corner. By the Addition Rule,

$$\mathsf{P}(E_1 \text{ or } E_2) = \mathsf{P}(E_1) + \mathsf{P}(E_2) - \mathsf{P}(E_1 \text{ and } E_2) = 0.05 + 0.08 - 0.004 = 0.126.$$

6 $${}_{32}P_4 = \frac{32!}{(32-4)!} = \frac{32!}{28!} = (29)(30)(31)(32) = 863{,}040.$$

7 There are $n = 11$ letters, with $n_1 = 1$ B's, $n_2 = 3$ e's, $n_3 = 1$ d, $n_4 = 2$ u's, $n_5 = 1$ t, $n_6 = 2$ n's, and $n_7 = 1$ g. Number of ways the letters can be written:

$$\frac{n!}{n_1!n_2!n_3!n_4!n_5!n_6!n_7!} = \frac{11!}{1!3!1!2!1!2!1!} = \frac{39{,}916{,}800}{24} = 1{,}663{,}200.$$

8 $$\frac{{}_3C_2 \cdot {}_{12}C_2 + {}_3C_3 \cdot {}_{12}C_1}{{}_{15}C_4} = \frac{(3)(66) + (1)(12)}{1365} = \frac{210}{1365} \approx 0.154.$$

9 The mean μ is:

$$\sum x\mathsf{P}(x) = 0(0.686) + 1(0.195) + 2(0.077) + 3(0.022) + 4(0.013) + 5(0.007) = 0.502.$$

The variance σ^2 is:

$$\sum (x-\mu)^2\mathsf{P}(x) = \sum (x - 0.502)^2\mathsf{P}(x)$$

$$= (0 - 0.502)^2(0.686) + (1 - 0.502)^2(0.195) + \cdots + (5 - 0.502)^2(0.007)$$
$$= 0.831996 \approx 0.832.$$

The standard deviation is $\sigma = \sqrt{0.831996} \approx 0.912$.

10a This is a binomial experiment with $n = 7$ independent trials, and if we define "success" to be "guessing the right answer," then the probability of success is $p = \frac{1}{4}$ for each trial. Thus the event of getting precisely five questions correct is the event of observing a total of $k = 5$ successes. The probability we seek is

$$\mathsf{P}(X = 5) = {}_7C_5\left(\frac{1}{4}\right)^5\left(1 - \frac{1}{4}\right)^{7-5} = \frac{7!}{(5!)(2!)}\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^2 \approx 0.0115.$$

10b Only getting 5, 6, or 7 questions right results in a score of at least 70%, and so we must find the probability that $X \geq 5$. The event $X \geq 5$ is the union of the events $X = 5$, $X = 6$, and $X = 7$, and since these are mutually exclusive events, the Addition Rule implies that

$$\mathsf{P}(X \geq 5) = \mathsf{P}\big(X = 5 \text{ or } X = 6 \text{ or } X = 7\big) = \mathsf{P}(X = 5) + \mathsf{P}(X = 6) + \mathsf{P}(X = 7)$$

We already found $\mathsf{P}(X = 5)$ in part (a). We calculate

$$\mathsf{P}(X = 6) = {}_7C_6\left(\frac{1}{4}\right)^6\left(1 - \frac{1}{4}\right)^{7-6} = \frac{7!}{(6!)(1!)}\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^1 \approx 0.00128,$$

and

$$\mathsf{P}(X = 7) = {}_7C_7\left(\frac{1}{4}\right)^7\left(1 - \frac{1}{4}\right)^{7-7} = \frac{7!}{(7!)(0!)}\left(\frac{1}{4}\right)^7\left(\frac{3}{4}\right)^0 \approx 6.10 \times 10^{-5}.$$

Therefore the probability of passing the quiz is

$$\mathsf{P}(X \geq 5) \approx 0.0115 + 0.00128 + 6.10 \times 10^{-5} \approx 0.0128.$$

11a This is a geometric experiment in which each trial consists of the drilling of a hole, "success" is defined to be the finding of a productive well, and the probability of success is $p = 0.18$ for each trial. The probability the fourth hole drilled is the first to yield a productive well is the probability the first success is observed after $k = 4$ trials have been performed:

$$\mathsf{P}(X = 4) = 0.18(1 - 0.18)^{4-1} = 0.18(0.82)^3 \approx 0.0992.$$

11b First find the probability the first success is observed after k trials have been performed, where $1 \leq k \leq 7$. This means calculating the probability that $X \leq 7$. The event $X \leq 7$ is the union of the seven events $X = 1, \ldots, X = 7$, and since these are mutually exclusive events the Addition Rule implies that

$$\mathsf{P}(X \leq 7) = \sum_{k=1}^{7}\mathsf{P}(X = k) = \sum_{k=1}^{7}0.18(1 - 0.18)^{k-1} = 0.18\sum_{k=1}^{7}(0.82)^{k-1}$$
$$= 0.18(1 + 0.82 + 0.82^2 + 0.82^3 + 0.82^4 + 0.82^5 + 0.82^6) \approx 0.751.$$

The probability of a failed search is the probability that the first success is *not* observed after 7 trials, which is $\mathsf{P}(X > 7)$. Since $\mathsf{P}(X \leq 7) + \mathsf{P}(X > 7) = 1$, we find that

$$\mathsf{P}(X > 7) = 1 - \mathsf{P}(X \leq 7) = 1 - 0.7507 \approx 0.249.$$

12a This is a Poisson experiment with parameter $\lambda = 0.6$, and so if the random variable X counts the number of hurricanes in a year, then

$$\mathsf{P}(X = k) = \frac{\lambda^k e^{-\lambda}}{k!} = \frac{0.6^k e^{-0.6}}{k!}$$

for $k = 0, 1, 2, \ldots$. Thus we have

$$\mathsf{P}(X = 1) = \frac{0.6e^{-0.6}}{1!} \approx 0.329.$$

12b We have

$$\begin{aligned}\mathsf{P}(X > 1) = 1 - \mathsf{P}(X \leq 1) &= 1 - [\mathsf{P}(X = 1) + \mathsf{P}(X = 0)] \\ &= 1 - \frac{0.6e^{-0.6}}{1!} - \frac{0.6^0 e^{-0.6}}{0!} = 1 - 0.3293 - 0.5488 \\ &= 0.1219.\end{aligned}$$