Math 103 Exam #2 Key (Summer II 2011)

1. $\dfrac{2x^3 - 11x^2 + 25}{x - 5} = 2x^2 - x - 5$

2a. $8r(r^2 + 3)$

2b. $3yz^3(5y^2 + 9yz - 12z^2)$

2c. $2(x+7)^3 - 3(x+7)^2 = (x+7)^2[2(x+7) - 3] = (x+7)^2(2x+11)$

3. $20 + 5s + 12t + 3st = 5(4+s) + 3t(4+s) = (4+s)(5+3t)$

4a. $r^2 - 2r - 35 = (r-7)(r+5)$

4b. $15p^2 + 24pq + 8q^2$ is prime

4c. $18c^2 - 98d^2 = 2(9c^2 - 49d^2) = 2\left[(3c)^2 - (7d)^2\right] = 2(3c - 7d)(3c + 7d)$

4d. $27y^3 + 8 = (3y)^3 + 2^3 = (3y+2)\left[(3y)^2 - (3y)(2) + 2^2\right] = (3y+2)(9y^2 - 6y + 4)$

4e. $x^4 - 625 = (x^2)^2 - 25^2 = (x^2 - 25)(x^2 + 25) = (x^2 - 5^2)(x^2 + 25) = (x-5)(x+5)(x^2+25)$

5a. $3x^2 + 10x + 3 = 0 \;\Rightarrow\; (3x+1)(x+3) = 0 \;\Rightarrow\; 3x + 1 = 0$ or $x + 3 = 0 \;\Rightarrow\; x = -1/3, -3$

5b. $6x^3 - 13x^2 - 5x = 0 \;\Rightarrow\; x(6x^2 - 13x - 5) = 0 \;\Rightarrow\; x(3x+1)(2x-5) = 0 \;\Rightarrow\; x = 0$ or $3x + 1 = 0$ or $2x - 5 = 0 \;\Rightarrow\; x = 0, -1/3, 5/2$

6a. $\dfrac{(v-6)(v+6)}{5(v+6)} = \dfrac{v-6}{5}$

6b. $\dfrac{(4x+1)(2x-3)}{(4x+3)(2x-3)} = \dfrac{4x+1}{4x+3}$

7a. $\dfrac{u^3v^2}{15u^2v^4} \cdot \dfrac{5v^{11}}{12u^4v^2} = \dfrac{1}{3} \cdot \dfrac{v^7}{12u^3} = \dfrac{v^7}{36u^3}$

7b. $\dfrac{(z-1)(z+1)}{6z} \cdot \dfrac{2}{-(z-1)} = \dfrac{z+1}{3z} \cdot \dfrac{1}{-1} = -\dfrac{z+1}{3z}$

7c. $\dfrac{(t-7)(t+7)}{(t+7)(t-3)} \cdot \dfrac{(t+5)(t+3)}{(t-7)(t+5)} = \dfrac{1}{t-3} \cdot \dfrac{t+3}{1} = \dfrac{t+3}{t-3}$

8a. $\dfrac{28}{12y} + \dfrac{27}{12y} = \dfrac{55}{12y}$

8b. $\dfrac{x-3}{(x-3)(x+2)} - \dfrac{x+2}{(x-3)(x+2)} = \dfrac{x-3-(x+2)}{(x-3)(x+2)} = -\dfrac{5}{(x-3)(x+2)}$

8c. $\dfrac{5x^2}{x(x-3)} + \dfrac{2(x-3)}{x(x-3)} + \dfrac{6}{x(x-3)} = \dfrac{5x^2+2(x-3)+6}{x(x-3)} = \dfrac{5x^2+2x}{x(x-3)} = \dfrac{x(5x+2)}{x(x-3)} = \dfrac{5x+2}{x-3}$

9. $\dfrac{4-\dfrac{1}{p}}{9+\dfrac{5}{p}} \cdot \dfrac{p}{p} = \dfrac{4p-1}{9p+5}$

10a. $2x(x+1)\cdot\left(2-\dfrac{5}{2x}\right) = 2x(x+1)\cdot\left(\dfrac{2x}{x+1}\right) \;\Rightarrow\; 4x(x+1)-5(x+1) = 4x^2 \;\Rightarrow\; 4x^2-x-5 = 4x^2 \;\Rightarrow\; x=-5$

10b. $2x(x+3)+4(x-3) = -24 \;\Rightarrow\; 2x^2+10x+12=0 \;\Rightarrow\; x^2+5x+6=0 \;\Rightarrow\; (x+3)(x+2) = 0 \;\Rightarrow\; x=-3,-2 \;\Rightarrow\; x=-2 \;\Rightarrow$ Solution set is $\{-2\}$.

11. $pqf\cdot\left(\dfrac{1}{p}+\dfrac{1}{q}\right) = \dfrac{1}{f}\cdot pqf \;\Rightarrow\; qf+pf=pq \;\Rightarrow\; pq-pf=qf \;\Rightarrow\; p(q-f)=qf \;\Rightarrow\; p=\dfrac{qf}{q-f}$