Math 103 Exam #3 Key (Spring 2014)

1a $\dfrac{(4x+1)(2x-3)}{(4x+3)(2x-3)} = \dfrac{4x+1}{4x+3}$

1b $\dfrac{(r-s)(r^2+rs+s^2)}{r-s} = r^2+rs+s^2$

2a $\dfrac{(a-1)(a+1)}{2(2a)} \cdot \dfrac{2}{-(a-1)} = \dfrac{a+1}{2a} \cdot \dfrac{1}{-1} = -\dfrac{a+1}{2a}$

2b $8(y-2) \cdot \dfrac{10}{3(y-2)} = 8 \cdot \dfrac{10}{3} = \dfrac{80}{3}$

3a $\dfrac{3z}{z(z-3)} - \dfrac{z-3}{z(z-3)} = \dfrac{3z-(z-3)}{z(z-3)} = \dfrac{2z+3}{z(z-3)}$

3b $\dfrac{4(w-3)}{(w+3)(w-3)} - \dfrac{w(w+3)}{(w+3)(w-3)} - \dfrac{18}{(w+3)(w-3)} = \dfrac{-w^2+w-30}{(w+3)(w-3)} = -\dfrac{(w-6)(w+5)}{(w+3)(w-3)}$

4 We have

$$\frac{q - \dfrac{q-3}{3}}{\dfrac{4}{9} + \dfrac{2}{3q}} \cdot \frac{9q}{9q} = \frac{9q^2 - 3q(q-3)}{4q+6} = \frac{3q(2q+3)}{2(2q+3)} = \frac{3q}{2}$$

5a Multiplying by p, we obtain

$$p^2 + 15 = -8p \quad\Rightarrow\quad p^2 + 8p + 15 = 0 \quad\Rightarrow\quad (p+5)(p+3) = 0 \quad\Rightarrow\quad p = -5, -3.$$

Solution set is $\{-5, -3\}$.

5b Multiplying by $t(6-3t)$, we obtain

$$5(6-3t) + 4t = 2t^2 \quad\Rightarrow\quad 2t^2 + 11t - 30 = 0 \quad\Rightarrow\quad (2t+15)(t-2) = 0,$$

so either $2t + 15 = 0$ or $t - 2 = 0$. Solving yields $t = -15/2$ or $t = 2$. But 2 is extraneous, so the solution set is $\{-15/2\}$.

6 We isolate n:

$$I = \frac{nE}{R+nr} \Rightarrow IR+Inr = nE \Rightarrow nE-Inr = IR \Rightarrow n(E-Ir) = IR \Rightarrow n = \frac{IR}{E-Ir}.$$

7

	Rate of Work	Time Worked	Fraction of Job Done
Krang	$\frac{1}{3}$	t	$\frac{t}{3}$
Shredder	$\frac{1}{5}$	t	$\frac{t}{5}$

Let t be the time it would take to complete the job. We get

$$\frac{t}{3} + \frac{t}{5} = 1 \Rightarrow 5t + 3t = 15 \Rightarrow t = \frac{15}{8} = 1\frac{7}{8} \text{ hours.}$$

8

	Rate of Work	Time Worked	Fraction of Job Done
Inlet	$\frac{1}{10}$	t	$\frac{t}{10}$
Outlet	$-\frac{1}{14}$	t	$-\frac{t}{14}$

Let t be the time it would take to complete the job. We get

$$\frac{t}{10} - \frac{t}{14} = 1 \Rightarrow 14t - 10t = 140 \Rightarrow t = \frac{140}{4} = 35 \text{ hours.}$$

9a The second equation gives $y = 4x + 1$, which we substitute into the first equation to get

$$3x + 2(4x + 1) = 13 \Rightarrow 11x = 11 \Rightarrow x = 1.$$

Thus we have $y = 4(1) + 1 = 5$. Solution is $(1, 5)$.

9b The second equation gives $y = 3x$, which we substitute into the first equation to get

$$\frac{1}{2}x + \frac{1}{3}(3x) = 3 \Rightarrow \frac{1}{2}x + x = 3 \Rightarrow \frac{3}{2}x = 3 \Rightarrow x = 2.$$

Putting this into either equation in the system yields $y = 6$. Solution is $(2, 6)$.

10 Don't overlook that w could be negative:

$$\sqrt[4]{w^{28}} = |w^7| = |w|^7.$$

(Either of the last two expressions are fine.)

11 We can assume that $m > 0$, since otherwise we would be taking the square root of a negative number:

$$(2m)^{-3/2} = [(2m)^3]^{-1/2} = (8m^3)^{-1/2} = \frac{1}{(8m^3)^{1/2}} = \frac{1}{\sqrt{8m^3}} = \frac{1}{2m\sqrt{2m}} = \frac{\sqrt{2m}}{4m^2}.$$

(Any of the last three expressions will be acceptable for this problem.)

12a $b^{2/3} \cdot b^{-1/6} = b^{2/3-1/6} = b^{1/2}$

12b $\dfrac{c^{1/6}h^{-5/6}}{(c^3h)^{1/3}} = \dfrac{c^{1/6}h^{-5/6}}{ch^{1/3}} = \dfrac{1}{c^{-1/6}c \cdot h^{5/6}h^{1/3}} = \dfrac{1}{c^{5/6}h^{7/6}}$