Math 103 Exam #4 Key (Spring 2012)

1a. $\sqrt{27} = 3\sqrt{3}$

1b. $\sqrt[3]{54w^3y^5} = 3wy\sqrt[3]{2y^2}$

1c. $\sqrt{\dfrac{r^3}{64}} = \dfrac{\sqrt{r^3}}{\sqrt{64}} = \dfrac{r\sqrt{r}}{8}$

1d. $3\sqrt{75} + 13\sqrt{48} = 3 \cdot 5\sqrt{3} + 13 \cdot 4\sqrt{3} = 15\sqrt{3} + 52\sqrt{3} = 67\sqrt{3}$

2. $(2\sqrt{3} + \sqrt{2})(2\sqrt{3} - \sqrt{2}) = 2\sqrt{3} \cdot 2\sqrt{3} - 2\sqrt{3} \cdot \sqrt{2} + \sqrt{2} \cdot 2\sqrt{3} - \sqrt{2} \cdot \sqrt{2} = 12 - 2 = 10.$

3a. $\dfrac{8}{\sqrt{24}} = \dfrac{8}{2\sqrt{6}} = \dfrac{4}{\sqrt{6}} = \dfrac{4}{\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{4\sqrt{6}}{6} = \dfrac{2\sqrt{6}}{3}$

3b. $\dfrac{1}{3 - \sqrt{b}} \cdot \dfrac{3 + \sqrt{b}}{3 + \sqrt{b}} = \dfrac{3 + \sqrt{b}}{9 - b}$

4a. $(\sqrt{9 - x})^2 = (x + 3)^2 \quad \Rightarrow \quad 9 - x = x^2 + 6x + 9 \quad \Rightarrow \quad x^2 + 7x = 0 \quad \Rightarrow \quad x(x + 7) = 0 \quad \Rightarrow$ $x = 0, -7$. But -7 is an extraneous solution, so the solution set is $\{0\}$.

4b. Cube both sides to get $2z - 1 = z - 11$, so $z = -10$. Solution set is $\{-10\}$.

5a. $(9 - 10i) - (5 + 3i) = 9 - 10i - 5 - 3i = 4 - 13i$

5b. $3i(4 + 7i) = 12i + 21i^2 = -21 + 12i$

5c. $\dfrac{3 - i}{1 - i} \cdot \dfrac{1 + i}{1 + i} = \dfrac{3 + 2i - i^2}{1 - i^2} = \dfrac{4 + 2i}{2} = 2 + i$

6a. Write as $x^2 - 2x - 4 = 0$, so

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)} = \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$$

6b. Write as $9x^2 - 6x + 7 = 0$, so

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(7)}}{2(9)} = \frac{6 \pm \sqrt{-216}}{18} = \frac{6 \pm 6i\sqrt{6}}{18} = \frac{1}{3} \pm \frac{\sqrt{6}}{3}i$$

7. Not a function. Domain is $[-3, 3]$ and range is $[-4, 4]$.

8. Is a function. Domain is $\{x : 4x + 2 \geq 0\} = \{x : x \geq -1/2\} = [-1/2, \infty)$.

9. $f(-3) = 3(-3) - 1 = -10$ and $g(-3) = (-3)^2 + 5(-3) = 9 - 15 = -6$.