

MATH 103 EXAM #3 KEY (SPRING 2012)

1a. $\frac{4x^2y + 12xy + 18x^3y^3}{8xy^2} = \frac{2xy(2x + 6 + 9x^2y^2)}{2xy \cdot 4y} = \frac{9x^2y^2 + 2x + 6}{4y}$

1b. $\frac{w^2 - 3w - 10}{w^2 + 5w + 6} = \frac{(w - 5)(w + 2)}{(w + 2)(w + 3)} = \frac{w - 5}{w + 3}$

2a. $\frac{x+1}{x^2 - 7x - 8} \cdot \frac{x^2 - x - 56}{x^2 + 9x + 14} = \frac{x+1}{(x-8)(x+1)} \cdot \frac{(x-8)(x+7)}{(x+7)(x+2)} = \frac{1}{1} \cdot \frac{1}{x+2} = \frac{1}{x+2}$

2b. $\frac{5a + 10b}{a^2 - 4b^2} \div \frac{a^3 + a^2b}{a^2 - 2ab} = \frac{5a + 10b}{a^2 - 4b^2} \cdot \frac{a^2 - 2ab}{a^3 + a^2b} = \frac{5(a + 2b)}{(a - 2b)(a + 2b)} \cdot \frac{a(a - 2b)}{a^2(a + b)} = \frac{5}{1} \cdot \frac{1}{a(a + b)} = \frac{5}{a^2 + ab}$

3a. $\frac{g+1}{2g} + \frac{3}{4g+8} = \frac{g+1}{2g} + \frac{3}{4(g+2)} = \frac{g+1}{2g} \cdot \frac{2(g+2)}{2(g+2)} + \frac{3}{4(g+2)} \cdot \frac{g}{g} = \frac{2(g+1)(g+2) + 3g}{4g(g+2)} = \frac{2g^2 + 9g + 4}{4g(g+2)}$
 $= \frac{(2g+1)(g+4)}{4g(g+2)}$

3b. $\frac{1}{r^2 - r - 6} - \frac{1}{r^2 - 4} = \frac{1}{(r-3)(r+2)} - \frac{1}{(r-2)(r+2)} = \frac{1}{(r-3)(r+2)} \cdot \frac{r-2}{r-2} - \frac{1}{(r-2)(r+2)} \cdot \frac{r-3}{r-3}$
 $= \frac{1}{(r-2)(r-3)(r+2)} - \frac{1}{(r-2)(r-3)(r+2)} = \frac{1}{(r-2)(r-3)(r+2)} = \frac{1}{(r-2)(r-3)(r+2)}$

4. $\frac{\frac{3}{u} - \frac{1}{u^2}}{5 + \frac{5}{u^2}} = \frac{\frac{3}{u} - \frac{1}{u^2}}{5 + \frac{5}{u^2}} \cdot \frac{u^2}{u^2} = \frac{3u - 1}{5u^2 + 5}$

5a. $\frac{y-3}{y+1} = \frac{y-6}{y+5} \Rightarrow (y+1)(y-6) = (y-3)(y+5) \Rightarrow y^2 - 5y - 6 = y^2 + 2y - 15 \Rightarrow -7y = -9 \Rightarrow y = \frac{9}{7}$.
 Solution set is $\{\frac{9}{7}\}$.

5b. $\frac{2}{c-5} = \frac{22}{2c^2 - 9c - 5} - \frac{3}{2c+1} \Rightarrow \frac{2}{c-5} = \frac{22}{(2c+1)(c-5)} - \frac{3}{2c+1} \Rightarrow 2(2c+1) = 22 - 3(c-5) \Rightarrow 4c + 2 = -3c + 37 \Rightarrow 7c = 35 \Rightarrow c = 5$. Solution is extraneous. Solution set is \emptyset .

6. $S = \frac{a}{1-r} \Rightarrow S(1-r) = a \Rightarrow S - Sr = a \Rightarrow Sr = S - a \Rightarrow r = \frac{S-a}{S}$

7.

	Rate	Time	Distance
Riding	12	$\frac{d}{12}$	d
Walking	3	$\frac{d}{3}$	d

He rides to the pub $36/60$ hr. faster, so $\frac{d}{12} + \frac{36}{60} = \frac{d}{3}$ is our equation (all time quantities should be in hours). This gives: $60\left(\frac{d}{12} + \frac{36}{60}\right) = \frac{d}{3} \cdot 60 \Rightarrow 5d + 36 = 20d \Rightarrow 15d = 36 \Rightarrow d = 36/15 = 2.4$ miles.

8.

	Rate of Work	Time Worked	Fraction of Job Done
Curly	$\frac{1}{10}$	t	$\frac{t}{10}$
Larry	$\frac{1}{8}$	t	$\frac{t}{8}$
Moe	$\frac{1}{6}$	t	$\frac{t}{6}$

Let t be the time it would take to complete the job. We get $\frac{t}{10} + \frac{t}{8} + \frac{t}{6} = 1 \Rightarrow 12t + 15t + 20t = 120 \Rightarrow 47t = 120 \Rightarrow t = 120/47 = 2\frac{26}{47}$ hours.

9a. The second equation gives $y = 3 - 2x$, which we substitute into the first equation to get $3x - 2(3 - 2x) = 7 \Rightarrow 3x - 6 + 4x = 7 \Rightarrow 7x = 13 \Rightarrow x = 13/7$. Putting this into either equation in the system yields $y = -5/7$. Solution is $(\frac{13}{7}, -\frac{5}{7})$.

9b. The second equation gives $y = 5x$, which we substitute into the first equation to get $\frac{1}{4}x - \frac{1}{5}(5x) = 9 \Rightarrow 5x - 4(5x) = 180 \Rightarrow -15x = 180 \Rightarrow x = -12$. Putting this into either equation in the system yields $y = -60$. Solution is $(-12, -60)$.

10. $\sqrt[3]{x^{15}} = x^5$.

11a. $4z^{5/3} \cdot 2z^{-7/2} = 8z^{5/3 + (-7/2)} = 8z^{-11/6} = \frac{8}{z^{11/6}}$

11b. $\left(\frac{x^{-1/3}}{x^{-2}}\right)^2 = (x^{-1/3} \cdot x^2)^2 = (x^{-1/3+2})^2 = (x^{5/3})^2 = x^{10/3}$