Math 103 Exam #1 Key (Fall 2014)

1a $5y - 2y + 10 = 25 \;\Rightarrow\; 3y = 15 \;\Rightarrow\; y = 5$

1b Multiply both sides by 2 to get $-5x+1 = -36$, from which comes $-5x = -37$ and finally $x = 37/5$

2 Multiply both sides by $\frac{10}{3}$ to get $\frac{10}{3}T = I - 12{,}000$. Thus $I = \frac{10}{3}T + 12{,}000$.

3 Let x be the number of moons that Ceti Alpha VI has, in which case Ceti Alpha V has $3x$ moons and Ceti Alpha VII has $2x+2$ moons. The total is 26 moons, so

$$x + 3x + (2x+2) = 26$$

is the equation. Solving gives $6x = 24$ and finally $x = 4$. That is, Ceti Alpha VI has 4 moons, Ceti Alpha V has 12 moons, and Ceti Alpha VII has 10 moons.

4 Let x be the original price of the desk. Then

$$x - 0.08x = 1007.40$$

is the equation. So $0.92x = 1007.40$, which solves to give $x = 1007.40/0.92 = 1095$. The original price is \$1095.

5 Let x be the number of liters of pure dye to be added. We equate liters of pure dye:

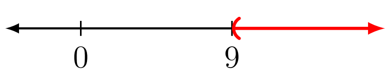
$$0.80x + 0.30(8) = 0.65(x+8).$$

From this we obtain

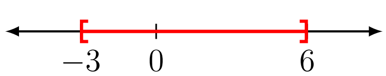
$$0.15x = 2.8 \;\Rightarrow\; x = \frac{2.8}{0.15} = 18\tfrac{2}{3}.$$

That is, $18\frac{2}{3}$ liters of the 80% dye solution should be added.

6a $-3x < -27 \;\Rightarrow\; x > 9 \;\Rightarrow\; (9, \infty)$



6b $-6 \le 2t \le 12 \;\Rightarrow\; -3 \le t \le 6 \;\Rightarrow\; [-3, 6]$



7a $x \le 15 \quad \text{and} \quad x \ge -7 \;\Rightarrow\; -7 \le x \le 15 \;\Rightarrow\; [-7, 15].$

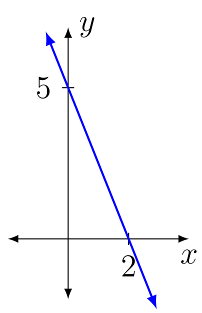
7b $3x < 24 \quad \text{or} \quad x > 10 \quad \Rightarrow \quad x < 8 \quad \text{or} \quad x > 10 \quad \Rightarrow \quad (-\infty, 8) \cup (10, \infty)$.

8 $7 - 3x = 16 \quad \text{or} \quad 7 - 3x = -16 \quad \Rightarrow \quad -3x = 9 \quad \text{or} \quad -3x = -23 \quad \Rightarrow \quad x = -3 \quad \text{or} \quad x = \frac{23}{3} \quad \Rightarrow \quad \left\{-3, \frac{23}{3}\right\}$

9a $3r - 1 > 11 \quad \text{or} \quad 3r - 1 < -11 \quad \Rightarrow \quad r > 4 \quad \text{or} \quad r < -\frac{10}{3} \quad \Rightarrow \quad \left(-\infty, -\frac{10}{3}\right) \cup (4, \infty)$

9b The inequality becomes $|z - 2| \leq -8$, and so the solution set is $\varnothing$ (i.e. there is no solution). An absolute value can never be less than 0.

10 x-intercept is $(2, 0)$, and y-intercept is $(0, 5)$.



11 Midpoint is at $\left(\dfrac{2+6}{2}, \dfrac{-3-8}{2}\right) = \left(4, -\dfrac{11}{2}\right)$

12 One line has equation $y = 2x - 3$ and thus slope 2, and the other line has equation $y = -\frac{1}{2}x + \frac{3}{2}$ and thus slope $-\frac{1}{2}$. Since the slopes are negative reciprocals, the lines are perpendicular.

13 Slope of the line is

$$m = \frac{10 - 6}{-8 - (-2)} = -\frac{4}{6} = -\frac{2}{3},$$

and so the equation is $y - 6 = -\frac{2}{3}(x + 2)$. Slope-intercept form:

$$y = -\frac{2}{3}x + \frac{14}{3}.$$

Standard form: $2x + 3y = 14$.

14 The line $5x + 3y = 12$, which can be written $y = -\frac{5}{3}x + 4$, has slope $-\frac{5}{3}$. Thus, the line whose equation we must find has point $(-2, 8)$ and slope $-\frac{5}{3}$ also, which gives us the equation

$y - 8 = -\frac{5}{3}(x+2)$ by the point-slope formula. Slope-intercept form and standard form are

$$y = -\tfrac{5}{3}x + \tfrac{14}{3} \quad \text{and} \quad 5x + 3y = 14,$$

respectively.

15a $-4r^{-2} = -\dfrac{4}{r^2}$

15b $(v^5)^{-4}v^8 = v^{-20}v^8 = v^{-12} = \dfrac{1}{v^{12}}$

15c $\dfrac{(2k)^2m^{-6}}{(km)^{-3}} = \dfrac{4k^2m^{-6}}{k^{-3}m^{-3}} = \dfrac{4k^2k^3}{m^6m^{-3}} = \dfrac{4k^5}{m^3}$