

MATH 103 EXAM #4 KEY (FALL 2013)

1a $\sqrt{500} = \sqrt{100 \cdot 5} = 10\sqrt{5}$

1b $\sqrt{121x^2y^7} = \sqrt{121 \cdot x^2 \cdot y^6 \cdot y} = 11xy^3\sqrt{y}$

1c $\sqrt[3]{54t^6z^4} = \sqrt[3]{27 \cdot 2 \cdot t^6 \cdot z^3 \cdot z} = 3t^2z\sqrt[3]{2z}$

1d $\sqrt{\frac{u^3}{81}} = \frac{\sqrt{u^2 \cdot u}}{\sqrt{81}} = \frac{u\sqrt{u}}{9}$

1e $3\sqrt{8} + 13\sqrt{72} - 3\sqrt{18} = 3 \cdot 2\sqrt{2} + 13 \cdot 6\sqrt{2} - 3 \cdot 3\sqrt{2} = 6\sqrt{2} + 78\sqrt{2} - 9\sqrt{2} = 75\sqrt{2}$

2 $(2\sqrt{3} + \sqrt{5})(2\sqrt{3} - \sqrt{5}) = 12 - 2\sqrt{15} + 2\sqrt{15} - 5 = 12 - 5 = 7$

3a $\frac{8}{\sqrt{24}} = \frac{8}{2\sqrt{6}} = \frac{8}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{8\sqrt{6}}{12} = \frac{2\sqrt{6}}{3}$

3b $\frac{3}{2 - \sqrt{a}} = \frac{3}{2 - \sqrt{a}} \cdot \frac{2 + \sqrt{a}}{2 + \sqrt{a}} = \frac{3(2 + \sqrt{a})}{4 - a}$

4a Solving:

$$\begin{aligned}\sqrt{9 - x} = x + 3 &\Rightarrow 9 - x = (x + 3)^2 \Rightarrow 9 - x = x^2 + 6x + 9 \\ &\Rightarrow x^2 + 7x = 0 \Rightarrow x(x + 7) = 0 \Rightarrow x = -7, 0.\end{aligned}$$

However, -7 is an extraneous solution since it gives $\sqrt{16} = -4$ when put into the original equation, so the solution set is $\{0\}$.

4b $\sqrt[3]{2y - 1} = \sqrt[3]{y + 13} \Rightarrow 2y - 1 = y + 13 \Rightarrow y = 14$, so the solution set is $\{14\}$.

5 $\sqrt{-7} \cdot \sqrt{-15} = i\sqrt{7} \cdot i\sqrt{15} = i^2 \cdot \sqrt{7} \cdot \sqrt{15} = -1 \cdot \sqrt{105} = -\sqrt{105}$

6 $115 \div 4 = 28\frac{3}{4}$, so $i^{115} = i^{4 \cdot 28 + 3} = i^{4 \cdot 28} \cdot i^3 = (i^4)^{28} \cdot i^3 = 1^{28} \cdot (-i) = -i$

7a $(9 + 11i) - (5 + 6i) = 9 + 11i - 5 - 6i = 4 + 5i$

7b $3i(4 - 9i) = 12i - 27i^2 = 12i + 27 = 27 + 12i$

7c $\frac{3 - i}{1 - i} = \frac{3 - i}{1 - i} \cdot \frac{1 + i}{1 + i} = \frac{(3 - i)(1 + i)}{(1 - i)(1 + i)} = \frac{3 + 3i - i - i^2}{1 - i^2} = \frac{3 + 2i + 1}{1 - (-1)} = \frac{4 + 2i}{2} = 2 + i$

8a Solving:

$$\begin{aligned}(x-3)(x+4) &= 2 \Rightarrow x^2 + x - 12 = 2 \Rightarrow x^2 + x - 14 = 0 \\ \Rightarrow x &= \frac{-1 \pm \sqrt{1^2 - 4(1)(-14)}}{2(1)} = \frac{-1 \pm \sqrt{57}}{2}\end{aligned}$$

8b Solving:

$$x^2 + 4x + 9 = 0 \Rightarrow x = \frac{-4 \pm \sqrt{4^2 - 4(1)(9)}}{2(1)} = \frac{-4 \pm \sqrt{-20}}{2} = \frac{-4 \pm 2i\sqrt{5}}{2} = -2 \pm \sqrt{5}i$$

9 Not a function. Domain is $[-3, 4]$ and range is $[-4, 5]$.

10 We have $2(0) - 1 < 3$ and $2(0) - 0 < 3$, which shows that the ordered pairs $(0, -1)$ and $(0, 0)$ are elements of the relation. Thus the relation is not a function since a function cannot have two ordered pairs with the same x -coordinate. The domain of the relation is $(-\infty, \infty)$.