Math 103 Exam #3 Key (Fall 2013)

1a $\dfrac{(4x+1)(2x-3)}{(4x+3)(2x-3)} = \dfrac{4x+1}{4x+3}$

1b $\dfrac{(r-s)(r^2+rs+s^2)}{r-s} = r^2+rs+s^2$

2a $\dfrac{(a-1)(a+1)}{2(2a)} \cdot \dfrac{2}{-(a-1)} = \dfrac{a+1}{2a} \cdot \dfrac{1}{-1} = -\dfrac{a+1}{2a}$

2b $8(y-2) \cdot \dfrac{10}{3(y-2)} = 8 \cdot \dfrac{10}{3} = \dfrac{80}{3}$

3a $\dfrac{3z}{z(z-3)} - \dfrac{z-3}{z(z-3)} = \dfrac{3z-(z-3)}{z(z-3)} = \dfrac{2z+3}{z(z-3)}$

3b $\dfrac{4(w-3)}{(w+3)(w-3)} - \dfrac{w(w+3)}{(w+3)(w-3)} - \dfrac{18}{(w+3)(w-3)} = \dfrac{-w^2+w-30}{(w+3)(w-3)} = -\dfrac{(w-6)(w+5)}{(w+3)(w-3)}$

4 We have

$$\frac{q-\dfrac{q-3}{3}}{\dfrac{4}{9}+\dfrac{2}{3q}} \cdot \frac{9q}{9q} = \frac{9q^2-3q(q-3)}{4q+6} = \frac{3q(2q+3)}{2(2q+3)} = \frac{3q}{2}$$

5a Multiplying by p, we obtain

$$p^2+15 = -8p \;\;\Rightarrow\;\; p^2+8p+15 = 0 \;\;\Rightarrow\;\; (p+5)(p+3) = 0 \;\;\Rightarrow\;\; p = -5, -3.$$

Solution set is $\{-5, -3\}$.

5b Multiplying by $t(6-3t)$, we obtain

$$5(6-3t)+4t = 2t^2 \;\;\Rightarrow\;\; 2t^2+11t-30 = 0 \;\;\Rightarrow\;\; (2t+15)(t-2) = 0,$$

so either $2t+15 = 0$ or $t-2 = 0$. Solving yields $t = -15/2$ or $t = 2$. But 2 is extraneous, so the solution set is $\{-15/2\}$.

6 We isolate n:

$$I = \frac{nE}{R+nr} \;\Rightarrow\; IR+Inr = nE \;\Rightarrow\; nE-Inr = IR \;\Rightarrow\; n(E-Ir) = IR \;\Rightarrow\; n = \frac{IR}{E-Ir}$$

7

	Rate	Time	Distance
Highway	90	$\frac{d}{90}$	d
Medicine Hat	45	$\frac{d-150}{45}$	$d-150$

Note that 50 minutes is 5/6 hour. Time driven in Medicine Hat is 5/6 hour less than time driven on highway, so

$$\frac{d-150}{45} = \frac{d}{90} - \frac{5}{6}.$$

Multiply by 90 to get

$$2(d-150) = d-15(5) \;\Rightarrow\; 2d-300 = d-75 \;\Rightarrow\; d = 225$$

as the distance driven on the highway. Total distance driven is $225 + (225 - 150) = 300$ km.

8

	Rate of Work	Time Worked	Fraction of Job Done
Inlet	$\frac{1}{10}$	t	$\frac{t}{10}$
Outlet	$-\frac{1}{14}$	t	$-\frac{t}{14}$

Let t be the time it would take to complete the job. We get

$$\frac{t}{10} - \frac{t}{14} = 1 \;\Rightarrow\; 14t - 10t = 140 \;\Rightarrow\; t = \frac{140}{4} = 35 \text{ hours.}$$

9a The second equation gives $y = 4x + 1$, which we substitute into the first equation to get

$$3x + 2(4x+1) = 13 \;\Rightarrow\; 11x = 11 \;\Rightarrow\; x = 1.$$

Thus we have $y = 4(1) + 1 = 5$. Solution is $(1, 5)$.

9b The second equation gives $y = 5x$, which we substitute into the first equation to get

$$\frac{1}{4}x - \frac{1}{5}(5x) = 9 \;\Rightarrow\; 5x - 4(5x) = 180 \;\Rightarrow\; -15x = 180 \;\Rightarrow\; x = -12.$$

Putting this into either equation in the system yields $y = -60$. Solution is $(-12, -60)$.

10 $\sqrt[6]{x^{30}} = |x^5| = |x|^5$

11 $(5u)^{-3/5} = [(5u)^3]^{-1/5} = (125u^3)^{-1/5} = \dfrac{1}{(125u^3)^{1/5}} = \dfrac{1}{\sqrt[5]{125u^3}}$

12a $b^{2/3} \cdot b^{-1/6} = b^{2/3-1/6} = b^{1/2}$

12b $\dfrac{c^{1/6}h^{-5/6}}{(c^3h)^{1/3}} = \dfrac{c^{1/6}h^{-5/6}}{ch^{1/3}} = \dfrac{1}{c^{-1/6}c \cdot h^{5/6}h^{1/3}} = \dfrac{1}{c^{5/6}h^{7/6}}$