

MATH 103 EXAM #1 KEY (FALL 2013)

**1a**  $2s - 1 = 6s - 5 \Rightarrow -4s = -4 \Rightarrow s = 1$

**1b**  $5x - 12 = 2x - 6 \Rightarrow 3x = 6 \Rightarrow x = 2$

**2**  $S = 2\pi rh + 2\pi r^2 \Rightarrow 2\pi rh = S - 2\pi r^2 \Rightarrow h = \frac{S - 2\pi r^2}{2\pi r}$ .

**3** Let  $x$  be the length of the equal sides, so that  $2x - 15$  is the length of the third side. The perimeter is the sum of the lengths of the sides, and it is given to be 53. Thus we have

$$x + x + (2x - 15) = 53,$$

or  $4x - 15 = 53$ . From this we get  $4x = 68$ , and finally  $x = 17$ . Therefore the equal sides are 17 cm long, and the third side is 19 cm.

**4** Let  $x$  be the amount invested at 5.5%, so that  $x - 4000$  is the amount invested at 4%. The interest from the 5.5% investment is  $0.055x$ , and the interest from the 4% investment is  $0.04(x - 4000)$ . Therefore

$$0.055x + 0.04(x - 4000) = 1900.$$

From this we have

$$0.095x - 160 = 1900 \Rightarrow 0.095x = 2060 \Rightarrow x = \frac{2060}{0.095} \Rightarrow x = 21,684.21$$

That is, \$21,684.21 was invested at 5.5%, and \$17,684.21 was invested at 4%.

**5** Let  $x$  be the number of gallons of pure dye to be added. We equate gallons of pure dye:

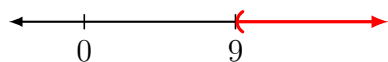
$$1.00x + 0.25(5) = 0.40(x + 5).$$

From this we obtain

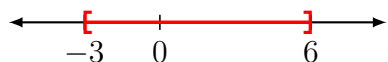
$$0.60x = 0.75 \Rightarrow x = \frac{0.75}{0.60} = 1.25.$$

That is, 1.25 gallons of pure dye should be added.

**6a**  $-3x < -27 \Rightarrow x > 9 \Rightarrow (9, \infty)$



**6b**  $-6 \leq 2t \leq 12 \Rightarrow -3 \leq t \leq 6 \Rightarrow [-3, 6]$



**7a**  $x \leq 15$  and  $x \geq -7 \Rightarrow -7 \leq x \leq 15 \Rightarrow [-7, 15]$ .

**7b**  $3x < 24$  or  $x > 10 \Rightarrow x < 8$  or  $x > 10 \Rightarrow (-\infty, 8) \cup (10, \infty)$ .

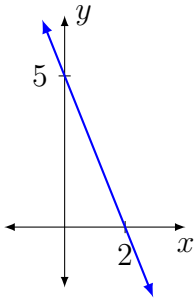
**8**  $7 - 3x = 16$  or  $7 - 3x = -16 \Rightarrow -3x = 9$  or  $-3x = -23 \Rightarrow x = -3$  or  $x = \frac{23}{3} \Rightarrow \left\{-3, \frac{23}{3}\right\}$

**9a**  $3r - 1 > 11$  or  $3r - 1 < -11 \Rightarrow r > 4$  or  $r < -\frac{10}{3} \Rightarrow (-\infty, -\frac{10}{3}) \cup (4, \infty)$

**9b**  $|y + 5| \leq 5 \Rightarrow -5 \leq y + 5 \leq 5 \Rightarrow -10 \leq y \leq 0 \Rightarrow [-10, 0]$

**9c** The solution set is  $\emptyset$ , which is to say there is no solution. An absolute value can never be less than 0.

**10**  $x$ -intercept is  $(2, 0)$ , and  $y$ -intercept is  $(0, 5)$ .



**11** Midpoint is at  $\left(\frac{2+6}{2}, \frac{-3-8}{2}\right) = \left(4, -\frac{11}{2}\right)$

**12** One line has equation  $y = 2x - 3$  and thus slope 2, and the other line has equation  $y = -\frac{1}{2}x + \frac{3}{2}$  and thus slope  $-\frac{1}{2}$ . Since the slopes are negative reciprocals, the lines are perpendicular.

**13** Slope of the line is

$$m = \frac{10 - 6}{-8 - (-2)} = -\frac{4}{6} = -\frac{2}{3},$$

and so the equation is  $y - 6 = -\frac{2}{3}(x + 2)$ . Slope-intercept form:

$$y = -\frac{2}{3}x + \frac{14}{3}.$$

Standard form:  $2x + 3y = 14$ .

**14** The line  $4x - y = 7$ , which can be written  $y = 4x - 7$ , has slope 4. Thus, the line whose equation we must find has point  $(-2, -3)$  and slope 4 also, which gives us the equation  $y + 3 = 4(x + 2)$  by the point-slope formula. Slope-intercept form:  $y = 4x + 5$ . Standard form:  $4x - y = -5$ .

**15a**  $2p^{-3} = \frac{2}{p^3}$

**15b**  $(v^5)^{-4}v^8 = v^{-20}v^8 = v^{-12} = \frac{1}{v^{12}}$

**15c**  $\frac{(2k)^2m^{-6}}{(km)^{-3}} = \frac{4k^2m^{-6}}{k^{-3}m^{-3}} = \frac{4k^2k^3}{m^6m^{-3}} = \frac{4k^5}{m^3}$