

MATH 103 EXAM #4 KEY (FALL 2012)

1a $\sqrt[4]{32} = 2\sqrt[4]{2}$

1b $\sqrt[3]{54w^3y^5} = 3wy\sqrt[3]{2y^2}$

1c $\sqrt{\frac{r^3}{64}} = \frac{\sqrt{r^3}}{\sqrt{64}} = \frac{r\sqrt{r}}{8}$

1d $\sqrt[12]{u^{16}} = u\sqrt[12]{u^4} = u\sqrt[3]{u}$

1e $4\sqrt{18r} - \sqrt{72r} = 4\sqrt{9 \cdot 2r} - \sqrt{36 \cdot 2r} = 4 \cdot 3\sqrt{2r} - 6\sqrt{2r} = 12\sqrt{2r} - 6\sqrt{2r} = 6\sqrt{2r}$

2a $(\sqrt{6} + \sqrt{2})(\sqrt{3} - \sqrt{2}) = \sqrt{18} - \sqrt{12} + \sqrt{6} - \sqrt{4} = 3\sqrt{2} - 2\sqrt{3} + \sqrt{6} - 2$

2b $(\sqrt{6} - \sqrt{2})^2 = (\sqrt{6} - \sqrt{2})(\sqrt{6} - \sqrt{2}) = 6 - \sqrt{12} - \sqrt{12} + 2 = 8 - 2\sqrt{12} = 8 - 4\sqrt{3}$

3a $\frac{8}{\sqrt{24}} = \frac{8}{2\sqrt{6}} = \frac{4}{\sqrt{6}} = \frac{4}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3}$

3b $\frac{1}{3 - \sqrt{b}} = \frac{1}{3 - \sqrt{b}} \cdot \frac{3 + \sqrt{b}}{3 + \sqrt{b}} = \frac{3 + \sqrt{b}}{9 - b}$

4a From $\sqrt{5k - 3} + 4 = 0$ we obtain $\sqrt{5k - 3} = -4$, which has no solution since a square root is never negative. Solution set is \emptyset .

4b $(\sqrt{9 - x})^2 = (x + 3)^2 \Rightarrow 9 - x = x^2 + 6x + 9 \Rightarrow x^2 + 7x = 0 \Rightarrow x(x + 7) = 0 \Rightarrow x = 0, -7$. But -7 is an extraneous solution, so the solution set is $\{0\}$.

4c Squaring both sides:

$$y^2 - 3y + 3 = (y - 1)^2 \Rightarrow y^2 - 3y + 3 = y^2 - 2y + 1 \Rightarrow y = 2.$$

Solution set is $\{2\}$.

5 $\sqrt{-27} \cdot \sqrt{-3} = i\sqrt{27} \cdot i\sqrt{3} = i^2 \cdot \sqrt{27}\sqrt{3} = -1 \cdot \sqrt{81} = -1 \cdot 9 = -9$

6a $9 - 10i + 5 + 3i = 14 - 7i$

6b $(4 + 3i)(-3 - 5i) = -12 - 20i - 9i - 15i^2 = -12 - 20i - 9i + 15 = 3 - 29i$

6c $\frac{2+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{2+2i+i+i^2}{1+i-i-i^2} = \frac{1+3i}{2} = \frac{1}{2} + \frac{3}{2}i$

6d $i^{71} = (i^4)^{17} \cdot i^3 = (1)^{17} \cdot (-i) = -i$

7a Write as $x^2 - 2x - 4 = 0$, so

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)} = \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$$

7b Write as $9x^2 - 6x + 7 = 0$, so

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(7)}}{2(9)} = \frac{6 \pm \sqrt{-216}}{18} = \frac{6 \pm 6i\sqrt{6}}{18} = \frac{1}{3} \pm \frac{\sqrt{6}}{3}i$$

8 Not a function. Domain is $[-5, 5]$ and range is $[-2, 2]$.

9 Is a function. Domain is $\{x : 3x - 1 \geq 0\} = \{x : x \geq 1/3\} = [1/3, \infty)$.

10 $f(-2) = 3(-2) - 1 = -7$ and $g(-2) = (-2)^2 - 5(-2) = 14$.