Math 103 Exam #3 Key (Fall 2012)

1a $\dfrac{2x^2-5x}{16x-40}=\dfrac{x(2x-5)}{8(2x-5)}=\dfrac{x}{8}$

1b $\dfrac{25m^2-n^2}{25m^2-10mn+n^2}=\dfrac{(5m-n)(5m+n)}{(5m-n)^2}=\dfrac{5m+n}{5m-n}$

2a $\dfrac{s^3t^2}{10s^2t^4}\div\dfrac{8s^4t^2}{5t^6}=\dfrac{s^3t^2}{10s^2t^4}\cdot\dfrac{5t^6}{8s^4t^2}=\dfrac{t^2}{16s^3}$

2b $\dfrac{t^2-49}{t^2+4t-21}\cdot\dfrac{t^2+8t+15}{t^2-2t-35}=\dfrac{(t-7)(t+7)}{(t+7)(t-3)}\cdot\dfrac{(t+5)(t+3)}{(t-7)(t+5)}=\dfrac{t+3}{t-3}$

3a $\dfrac{5}{12x^5y^2}+\dfrac{5}{18x^4y^5}=\dfrac{5}{12x^5y^2}\cdot\dfrac{3y^3}{3y^3}+\dfrac{5}{18x^4y^5}\cdot\dfrac{2x}{2x}=\dfrac{15y^3}{36x^5y^5}+\dfrac{10x}{36x^5y^5}=\dfrac{15y^3+10x}{36x^5y^5}$

3b $\dfrac{5x}{x+3}+\dfrac{x+2}{x}-\dfrac{6}{x^2+3x}=\dfrac{5x^2}{x(x+3)}+\dfrac{(x+2)(x+3)}{x(x+3)}-\dfrac{6}{x(x+3)}=\dfrac{x(6x+5)}{x(x+3)}=\dfrac{6x+5}{x+3}$

4 We have

$$\frac{1-\dfrac{2}{3x}}{9-\dfrac{4}{x^2}}=\frac{1-\dfrac{2}{3x}}{9-\dfrac{4}{x^2}}\cdot\frac{3x^2}{3x^2}=\frac{3x^2-2x}{27x^2-12}=\frac{x(3x-2)}{3(3x-2)(3x+2)}=\frac{x}{9x+6}$$

5a Multiplying by $(7-a)(a+3)$, we obtain

$$\frac{4}{7-a}=\frac{2a}{a+3}\ \Rightarrow\ 4(a+3)=2a(7-a)\ \Rightarrow\ 2a^2-10a+12=0$$
$$\Rightarrow\ a^2-5a+6=0\ \Rightarrow\ (a-3)(a-2)=0\ \Rightarrow\ a=2,3.$$

Solution set is $\{2,3\}$.

5b Multiplying by $z(z+2)$ yields $3(z+2)+z^2=4$, or $z^2+3z+2=0$. Factoring, we get $(z+2)(z+1)=0$, which has solutions $z=-2,-1$. But -2 is extraneous, so the solution set is $\{-1\}$.

6 We isolate r: $I=\dfrac{nE}{R+nr}\ \Rightarrow\ IR+Inr=nE\ \Rightarrow\ Inr=nE-IR\ \Rightarrow\ r=\dfrac{nE-IR}{In}$.

7

	Rate	Time	Distance
Riding	12	$\frac{d}{12}$	d
Walking	3	$\frac{d}{3}$	d

He rides to the pub 36/60 hr. faster, so $\frac{d}{12} + \frac{36}{60} = \frac{d}{3}$ is our equation (all time quantities should be in hours). This gives:

$$60\left(\frac{d}{12} + \frac{36}{60}\right) = \frac{d}{3} \cdot 60 \quad \Rightarrow \quad 5d + 36 = 20d \quad \Rightarrow \quad 15d = 36 \quad \Rightarrow \quad d = 36/15 = 2.4 \text{ miles.}$$

8

	Rate of Work	Time Worked	Fraction of Job Done
Pipe	$\frac{1}{12}$	t	$\frac{t}{12}$
Hose	$\frac{1}{30}$	t	$\frac{t}{30}$

Let t be the time it would take to complete the job. We get

$$\frac{t}{12} + \frac{t}{30} = 1 \quad \Rightarrow \quad 5t + 2t = 60 \quad \Rightarrow \quad t = 60/7 = 8\frac{4}{7} \text{ hours.}$$

9a The second equation gives $y = 4x + 1$, which we substitute into the first equation to get

$$3x + 2(4x + 1) = 13 \quad \Rightarrow \quad 11x = 11 \quad \Rightarrow \quad x = 1.$$

Thus we have $y = 4(1) + 1 = 5$ Solution is $(1, 5)$.

9b The second equation gives $y = 5x$, which we substitute into the first equation to get

$$\frac{1}{4}x - \frac{1}{5}(5x) = 9 \quad \Rightarrow \quad 5x - 4(5x) = 180 \quad \Rightarrow \quad -15x = 180 \quad \Rightarrow \quad x = -12.$$

Putting this into either equation in the system yields $y = -60$. Solution is $(-12, -60)$.

10 $\sqrt{(-q)^2} = |-q| = |q|$

11 $(2m)^{-2/3} = [(2m)^2]^{-1/3} = (4m^2)^{-1/3} = \dfrac{1}{(4m^2)^{1/3}} = \dfrac{1}{\sqrt[3]{4m^2}}$

12a $x^{2/5} \cdot x^{-1/4} = x^{2/5-1/4} = x^{3/20}$

12b $\dfrac{c^{1/6}h^{-5/6}}{(c^3h)^{1/3}} = \dfrac{c^{1/6}h^{-5/6}}{ch^{1/3}} = \dfrac{1}{c^{-1/6}c \cdot h^{5/6}h^{1/3}} = \dfrac{1}{c^{5/6}h^{7/6}}$