Math 103 Exam #2 Key (Fall 2012)

1 $(q^4 - 2q^2 + 10) + (3q^4 + 5q^2 - 5) = 4q^4 + 3q^2 + 5.$

2 $(4y^2 - 6y - 7) - (-3y^2 + 5y - 9) = 4y^2 - 6y - 7 + 3y^2 - 5y + 9 = 7y^2 - 11y + 2.$

3a $4w^2(-7w^4) = -28w^6.$

3b $(6c - d)(2c + 3d) = 12c^2 + 18cd - 2cd - 3d^2 = 12c^2 + 16cd - 3d^2.$

3c $(3z - 4)(4z^2 + 5z - 6) = 12z^3 + 15z^2 - 18z - 16z^2 - 20z + 24 = 12z^3 - z^2 - 38z + 24.$

4a Apply long division to obtain $\dfrac{x^3 + 3x^2 - 4}{x + 2} = x^2 + x - 2$, as shown below:

$$
\begin{array}{r}
x^2 + x - 2 \\
\hline
x + 2 \,\big)\; x^3 + 3x^2 \quad\quad - 4 \\
\underline{-x^3 - 2x^2} \\
\quad x^2 \\
\quad \underline{-x^2 - 2x} \\
\quad\quad -2x - 4 \\
\quad\quad \underline{2x + 4} \\
\quad\quad\quad 0
\end{array}
$$

4b Apply long division to obtain $\dfrac{3t^4 + 5t^3 - 8t^2 - 13t + 2}{t^2 - 5} = 3t^2 + 5t + 7 + \dfrac{12t + 37}{t^2 - 5}$, as shown:

$$
\begin{array}{r}
3t^2 + 5t + 7 \\
\hline
t^2 - 5 \,\big)\; 3t^4 + 5t^3 - 8t^2 - 13t + 2 \\
\underline{-3t^4 \quad\quad + 15t^2} \\
\quad\quad 5t^3 + 7t^2 - 13t \\
\quad\quad \underline{-5t^3 \quad\quad + 25t} \\
\quad\quad\quad\quad 7t^2 + 12t + 2 \\
\quad\quad\quad\quad \underline{-7t^2 \quad\quad + 35} \\
\quad\quad\quad\quad\quad\quad 12t + 37
\end{array}
$$

5a $10m^5 - 8m^2 - 4m^2 = 2m^2(5m^3 - 4m - 2).$

5b $2(5 - x)^3 - 3(5 - x)^2 = (5 - x)^2[2(5 - x) - 3] = (5 - x)^2(10 - 2x - 3) = (5 - x)^2(7 - 2x).$

6 $2xy + 3y + 2x + 3 = (2xy + 3y) + (2x + 3) = y(2x + 3) + (2x + 3) = (2x + 3)(y + 1).$

7a $n^2 - 3nq - 15q^2$ is prime.

7b $13r^3 + 39r^2 - 52r = 13r(r^2 + 3r - 4) = 13r(r + 4)(r - 1)$.

7c $16 - (a + 3z)^2 = 4^2 - (a + 3z)^2 = [4 - (a + 3z)][4 + (a + 3z)] = (4 - a - 3z)(4 + a + 3z)$.

7d $8v^3 + 1 = (2v)^3 + 1^3 = (2v + 1)[(2v)^2 - (2v)(1) + 1^2] = (2v + 1)(4v^2 - 2v + 1)$.

7e $u^3 + u^2 - w^3 - w^2 = (u^3 - w^3) + (u^2 - w^2) = (u - w)(u^2 + uw + w^2) + (u - w)(u + w) = (u - w)(u^2 + uw + w^2 + u + w)$.

8a $x^2 - x - 12 = 0 \Rightarrow (x - 4)(x + 3) = 0 \Rightarrow x - 4 = 0$ or $x + 3 = 0 \Rightarrow x = 4$ or $x = -3 \Rightarrow \{-3, 4\}$ is the solution set.

8b $y^2 + 6y - 16 = -21 \Rightarrow y^2 + 6y + 5 = 0 \Rightarrow (y + 5)(y + 1) = 0 \Rightarrow y + 5 = 0$ or $y + 1 = 0 \Rightarrow y = -5$ or $y = -1 \Rightarrow \{-5, -1\}$ is the solution set.

8c $z^3 - 6z^2 + 8z = 0 \Rightarrow z(z - 2)(z - 4) = 0 \Rightarrow z = 0, 2, 4 \Rightarrow \{0, 2, 4\}$ is the solution set.