Math 103 Exam #1 Key (Fall 2012)

1a $2s - 1 = 6s - 5 \quad \Rightarrow \quad -4s = -4 \quad \Rightarrow \quad s = 1$

1b $5x - 12 = 2x - 6 \quad \Rightarrow \quad 3x = 6 \quad \Rightarrow \quad x = 2$

2 From $V = \ell w h$ we obtain $w = \dfrac{V}{\ell h}$.

3 Let x be the length of the equal sides, so that $2x - 15$ is the length of the third side. The perimeter is the sum of the lengths of the sides, and it is given to be 53. Thus we have

$$x + x + (2x - 15) = 53,$$

or $4x - 15 = 53$. From this we get $4x = 68$, and finally $x = 17$. Therefore the equal sides are 17 cm long, and the third side is 19 cm.

4 Let x be the amount invested at 5%, so that $x - 4000$ is the amount invested at 3%. The interest from the 5% investment is $0.05x$, and the interest from the 3% investment is $0.03(x - 4000)$. Therefore

$$0.05x + 0.03(x - 4000) = 900.$$

From this we have

$$0.08x - 120 = 900 \quad \Rightarrow \quad 0.08x = 1020 \quad \Rightarrow \quad x = \frac{1020}{0.08} \quad \Rightarrow \quad x = 12,750.$$

That is, \$12,750 is invested at 5%, and \$8,750 is invested at 3%.

5 Let x be the number of kilograms of pistachios to be added. We equate dollar values:

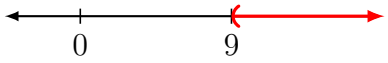
$$2(40) + 5x = 4(x + 40).$$

From this we obtain

$$80 + 5x = 4x + 160 \quad \Rightarrow \quad x = 80.$$

That is, 80 kilograms of pistachios should be mixed with the peanuts.

6a $-3x < -27 \quad \Rightarrow \quad x > 9 \quad \Rightarrow \quad (9, \infty)$



6b $-6 \le 2t \le 12 \quad \Rightarrow \quad -3 \le t \le 6 \quad \Rightarrow \quad [-3, 6]$

7a $x \le 15$ and $x \ge -7 \quad \Rightarrow \quad -7 \le x \le 15 \quad \Rightarrow \quad [-7, 15]$.

7b $3x < 24 \quad \text{or} \quad x > 10 \quad \Rightarrow \quad x < 8 \quad \text{or} \quad x > 10 \quad \Rightarrow \quad (-\infty, 8) \cup (10, \infty)$.

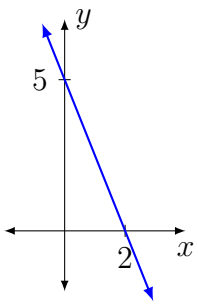
8 $7 - 3x = 16 \quad \text{or} \quad 7 - 3x = -16 \quad \Rightarrow \quad -3x = 9 \quad \text{or} \quad -3x = -23 \quad \Rightarrow \quad x = -3 \quad \text{or} \quad x = \frac{23}{3} \quad \Rightarrow \quad \left\{-3, \frac{23}{3}\right\}$

9a $3r - 1 > 11 \quad \text{or} \quad 3r - 1 < -11 \quad \Rightarrow \quad r > 4 \quad \text{or} \quad r < -\frac{10}{3} \quad \Rightarrow \quad \left(-\infty, -\frac{10}{3}\right) \cup (4, \infty)$

9b $|y + 5| \leq 5 \quad \Rightarrow \quad -5 \leq y + 5 \leq 5 \quad \Rightarrow \quad -10 \leq y \leq 0 \quad \Rightarrow \quad [-10, 0]$

9c The solution set is $(-\infty, \infty)$, which is all real numbers, since an absolute value is always greater than or equal to 0.

10 x-intercept is $(2, 0)$, and y-intercept is $(0, 5)$.



11 Midpoint is at $\left(\dfrac{2+6}{2}, \dfrac{-3-8}{2}\right) = \left(4, -\dfrac{11}{2}\right)$

12 One line has equation $y = 2x - 3$ and thus slope 2, and the other line has equation $y = -\frac{1}{2}x + \frac{3}{2}$ and thus slope $-\frac{1}{2}$. Since the slopes are negative reciprocals, the lines are perpendicular.

13 Slope of the line is $m = \dfrac{10 - (-3)}{-8 - (-2)} = -\dfrac{13}{6}$, and so equation is $y - (-3) = -\frac{13}{6}(x + 2)$. Slope-intercept form: $y = -\frac{13}{6}x - \frac{22}{3}$; standard form: $13x + 6y = -44$.

14 The line $4x - y = 7$, which can be written $y = 4x - 7$, has slope 4. Thus, the line whose equation we must find has point $(-2, -3)$ and slope 4 also, which gives us the equation $y + 3 = 4(x + 2)$ by the point-slope formula. Slope-intercept form: $y = 4x + 5$. Standard form: $4x - y = -5$.

15a $2y^{-5} = \dfrac{2}{y^5}$

15b $(t^5)^{-3}t^7 = t^{-15}t^7 = t^{-8} = \dfrac{1}{t^8}$

15c $\dfrac{(2k)^2m^{-6}}{(km)^{-3}} = \dfrac{4k^2m^{-6}}{k^{-3}m^{-3}} = \dfrac{4k^2k^3}{m^6m^{-3}} = \dfrac{4k^5}{m^3}$