Math 103 Exam #4 Key (Fall 2011)

1a. $\sqrt{500} = \sqrt{100 \cdot 5} = 10\sqrt{5}$

1b. $\sqrt{121x^2y^7} = \sqrt{121 \cdot x^2 \cdot y^6 \cdot y} = 11xy^3\sqrt{y}$

1c. $\sqrt[3]{54t^6z^4} = \sqrt[3]{27 \cdot 2 \cdot t^6 \cdot z^3 \cdot z} = 3t^2z\sqrt[3]{2z}$

1d. $\sqrt{\dfrac{u^3}{81}} = \dfrac{\sqrt{u^2 \cdot u}}{\sqrt{81}} = \dfrac{u\sqrt{u}}{9}$

1e. $3\sqrt{8} + 13\sqrt{72} - 3\sqrt{18} = 3 \cdot 2\sqrt{2} + 13 \cdot 6\sqrt{2} - 3 \cdot 3\sqrt{2} = 6\sqrt{2} + 78\sqrt{2} - 9\sqrt{2} = 75\sqrt{2}$

2. $\left(2\sqrt{3} + \sqrt{5}\,\right)\left(2\sqrt{3} - \sqrt{5}\,\right) = 12 - 2\sqrt{15} + 2\sqrt{15} - 5 = 12 - 5 = 7$

3a. $\dfrac{8}{\sqrt{24}} = \dfrac{8}{2\sqrt{6}} = \dfrac{8}{2\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{8\sqrt{6}}{12} = \dfrac{2\sqrt{6}}{3}$

3b. $\dfrac{3}{2 - \sqrt{a}} = \dfrac{3}{2 - \sqrt{a}} \cdot \dfrac{2 + \sqrt{a}}{2 + \sqrt{a}} = \dfrac{3(2 + \sqrt{a}\,)}{4 - a}$

4a. $\sqrt{9 - x} = x + 3 \;\Rightarrow\; 9 - x = (x+3)^2 \;\Rightarrow\; 9 - x = x^2 + 6x + 9 \;\Rightarrow\; x^2 + 7x = 0 \;\Rightarrow\; x(x+7) = 0 \;\Rightarrow\; x = -7, 0.$ However, -7 is an extraneous solution since it gives $\sqrt{16} = -4$ when put into the original equation, so the solution set is $\{0\}$.

4b. $\sqrt[3]{2y - 1} = \sqrt[3]{y + 13} \;\Rightarrow\; 2y - 1 = y + 13 \;\Rightarrow\; y = 14$, so the solution set is $\{14\}$.

5. $\sqrt{-7} \cdot \sqrt{-15} = i\sqrt{7} \cdot i\sqrt{15} = i^2 \cdot \sqrt{7} \cdot \sqrt{15} = -1 \cdot \sqrt{105} = -\sqrt{105}$

6. $91 \div 4 = 22\frac{3}{4}$, so $i^{91} = i^{4 \cdot 22 + 3} = i^{4 \cdot 22} \cdot i^3 = (i^4)^{22} \cdot i^3 = 1^{88} \cdot (-i) = -i$

7a. $(9 + 11i) - (5 + 6i) = 9 + 11i - 5 - 6i = 4 + 5i$

7b. $3i(4 - 9i) = 12i - 27i^2 = 12i + 27 = 27 + 12i$

7c. $\dfrac{3 - i}{1 - i} = \dfrac{3 - i}{1 - i} \cdot \dfrac{1 + i}{1 + i} = \dfrac{(3 - i)(1 + i)}{(1 - i)(1 + i)} = \dfrac{3 + 3i - i - i^2}{1 - i^2} = \dfrac{3 + 2i + 1}{1 - (-1)} = \dfrac{4 + 2i}{2} = 2 + i$

8a. $(x - 3)(x + 4) = 2 \;\Rightarrow\; x^2 + x - 12 = 2 \;\Rightarrow\; x^2 + x - 14 = 0 \;\Rightarrow\; x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-14)}}{2(1)} = \dfrac{-1 \pm \sqrt{57}}{2}$

8b. $x^2 + 4x + 9 = 0 \;\Rightarrow\; x = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(9)}}{2(1)} = \dfrac{-4 \pm \sqrt{-20}}{2} = \dfrac{-4 \pm 2i\sqrt{5}}{2} = -2 \pm \sqrt{5}i$

9. Yes Virginia, it is a function. Domain is $[-2, 2]$ and range is $[0, 3]$.

10. It helps to solve for y: $y = \pm\sqrt[6]{x}$. This shows that the relation is not a function, since, for example, the ordered pairs $(64, 2)$ and $(64, -2)$ belong to the relation. The domain is $[0, \infty)$.