Math 103 Exam #3 Key (Fall 2011)

1a. $\dfrac{v^2-36}{5v+30} = \dfrac{(v-6)(v+6)}{5(v+6)} = \dfrac{v-6}{5}$

1b. $\dfrac{8x^2-10x-3}{8x^2-6x-9} = \dfrac{(4x+1)(2x-3)}{(4x+3)(2x-3)} = \dfrac{4x+1}{4x+3}$

2a. $\dfrac{u^3v^2}{15u^2v^4} \div \dfrac{12u^4v^2}{5v^{11}} = \dfrac{u^3v^2}{15u^2v^4} \cdot \dfrac{5v^{11}}{12u^4v^2} = \dfrac{1}{3} \cdot \dfrac{v^7}{12u^3} = \dfrac{v^7}{36u^3}$

2b. $\dfrac{z^2-1}{6z} \cdot \dfrac{2}{1-z} = \dfrac{(z-1)(z+1)}{6z} \cdot \dfrac{2}{-(z-1)} = \dfrac{z+1}{3z} \cdot \dfrac{1}{-1} = -\dfrac{z+1}{3z}$

3a. $\dfrac{x-3}{(x-3)(x+2)} - \dfrac{x+2}{(x-3)(x+2)} = \dfrac{x-3-(x+2)}{(x-3)(x+2)} = -\dfrac{5}{(x-3)(x+2)}$

3b. $\dfrac{5x^2}{x(x-3)} + \dfrac{2(x-3)}{x(x-3)} + \dfrac{6}{x(x-3)} = \dfrac{5x^2+2(x-3)+6}{x(x-3)} = \dfrac{5x^2+2x}{x(x-3)} = \dfrac{x(5x+2)}{x(x-3)} = \dfrac{5x+2}{x-3}$

4. $\dfrac{4-\dfrac{1}{p}}{9+\dfrac{5}{p}} = \dfrac{4-\dfrac{1}{p}}{9+\dfrac{5}{p}} \cdot \dfrac{p}{p} = \dfrac{4p-1}{9p+5}$

5a. $2x(x+1)\cdot\left(2-\dfrac{5}{2x}\right) = 2x(x+1)\cdot\left(\dfrac{2x}{x+1}\right) \;\Rightarrow\; 4x(x+1)-5(x+1) = 4x^2 \;\Rightarrow\; 4x^2-x-5 = 4x^2 \;\Rightarrow\; x = -5$

5b. $2x(x+3)+4(x-3) = -24 \;\Rightarrow\; 2x^2+10x+12 = 0 \;\Rightarrow\; x^2+5x+6 = 0 \;\Rightarrow\; (x+3)(x+2) = 0 \;\Rightarrow\; x = -3, -2 \;\Rightarrow\; x = -2 \;\Rightarrow$ Solution set is $\{-2\}$.

6. $I = \dfrac{nE}{R+nr} \;\Rightarrow\; I(R+nr) = nE \;\Rightarrow\; IR+Inr = nE \;\Rightarrow\; In\cdot r = nE-IR \;\Rightarrow\; r = \dfrac{nE-IR}{nI}$

7.

	Rate	Time	Distance
Riding	12	$\dfrac{d}{12}$	d
Walking	3	$\dfrac{d}{3}$	d

He rides to the pub 36/60 hr. faster, so $\dfrac{d}{12} + \dfrac{36}{60} = \dfrac{d}{3}$ is our equation (all time quantities should be in hours). This gives: $60\left(\dfrac{d}{12} + \dfrac{36}{60}\right) = \dfrac{d}{3}\cdot 60 \;\Rightarrow\; 5d+36 = 20d \;\Rightarrow\; 15d = 36 \;\Rightarrow\; d = 36/15 = 2.4$ miles.

8.

	Rate of Work	Time Worked	Fraction of Job Done
Inlet	$\frac{1}{10}$	t	$\frac{t}{10}$
Outlet	$-\frac{1}{13}$	t	$-\frac{t}{13}$

Let t be the time it would take to fill the vat with inlet and outlet both open. We get $\frac{t}{10} - \frac{t}{13} = 1 \;\Rightarrow\; 13t - 10t = 130 \;\Rightarrow\; 3t = 130 \;\Rightarrow\; t = 43\frac{1}{3}$ hours.

9a. The second equation gives $y = 3 - 2x$, which we substitute into the first equation to get $3x - 2(3 - 2x) = 7 \;\Rightarrow\; 3x - 6 + 4x = 7 \;\Rightarrow\; 7x = 13 \;\Rightarrow\; x = 13/7$. Putting this into either equation in the system yields $y = -5/7$. Solution is $\left(\frac{13}{7}, -\frac{5}{7}\right)$.

9b. The second equation gives $y = 5x$, which we substitute into the first equation to get $\frac{1}{4}x - \frac{1}{5}(5x) = 9 \;\Rightarrow\; 5x - 4(5x) = 180 \;\Rightarrow\; -15x = 180 \;\Rightarrow\; x = -12$. Putting this into either equation in the system yields $y = -60$. Solution is $(-12, -60)$.

10. $81^{-3/4} = \dfrac{1}{81^{3/4}} = \dfrac{1}{(81^{1/4})^3} = \dfrac{1}{\left(\sqrt[4]{81}\right)^3} = \dfrac{1}{3^3} = \dfrac{1}{27}.$

11a. $r^{-8/9} \cdot r^{19/9} = r^{-8/9+19/9} = r^{11/9}$

11b. $\dfrac{m^{3/4}n^{-1/4}}{(m^2n)^{1/2}} = \dfrac{m^{3/4}n^{-1/4}}{(m^2)^{1/2} \cdot n^{1/2}} = \dfrac{m^{3/4}n^{-1/4}}{m \cdot n^{1/2}} = \dfrac{1}{m^{-3/4} \cdot m \cdot n^{1/2} \cdot n^{1/4}} = \dfrac{1}{m^{1/4}n^{3/4}}$