A1) For any $a, b \in \mathbb{Z}$, $a + b \in \mathbb{Z}$ and $a \cdot b \in \mathbb{Z}$
A2) For any $a, b \in \mathbb{Q}$, $a + b \in \mathbb{Q}$ and $a \cdot b \in \mathbb{Q}$
A3) For any $a, b \in \mathbb{R}$, $a + b \in \mathbb{R}$ and $a \cdot b \in \mathbb{R}$
A4) For any $a, b, c \in \mathbb{R}$, $a + (b + c) = (a + b) + c$
A5) For any $a, b, c \in \mathbb{R}$, $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
A6) For any $a \in \mathbb{R}$, $a + 0 = a = 0 + a$ and $a \cdot 1 = a = 1 \cdot a$
A7) For any $a, b \in \mathbb{R}$, $a + b = b + a$
A8) For any $a, b \in \mathbb{R}$, $a \cdot b = b \cdot a$
A9) For any $a, b \in \mathbb{R}$, if $a \cdot b = 0$ then either $a = 0$ or $b = 0$

1. [25 pts.] Determine which of the five properties of a commutative group hold for the mathematical system consisting of the set of whole numbers $\{0, 1, 2, 3, \ldots\}$ under the operation of subtraction. Use the axioms given above where appropriate.

2. [25 pts.] The system consisting of the set of whole numbers $\{0, 1, 2, 3, \ldots\}$ and the binary operation $\ominus$ defined as follows: $a \ominus b = |a - b|$ (i.e. the absolute value of $a - b$). Use the axioms given above where appropriate, and assume that the absolute value of an integer is an integer.

3. [25 pts.] For the given mathematical system determine which of the five properties of a commutative group hold. If a property holds, explain why. If a property fails, give a counterexample.

$\otimes$	$\sqcup$	$\vee$	$\cap$
$\sqcup$	$\sqcup$	$\vee$	$\cap$
$\vee$	$\vee$	$\sqcup$	$\cap$
$\cap$	$\cap$	$\vee$	$\sqcup$

4. [25 pts.] For the given mathematical system determine which of the five properties of a commutative group hold. If a property holds, explain why. If a property fails, give a counterexample.

$\boxdot$	$\ltimes$	$\curlyvee$	$>$	$\bowtie$	$\perp$
$\ltimes$	$\bowtie$	$\perp$	$\ltimes$	$\curlyvee$	$\curlyvee$
$\curlyvee$	$\perp$	$\ltimes$	$\curlyvee$	$\ltimes$	$\bowtie$
$>$	$\ltimes$	$\curlyvee$	$>$	$\bowtie$	$\perp$
$\bowtie$	$\curlyvee$	$\ltimes$	$\bowtie$	$\perp$	$>$
$\perp$	$\curlyvee$	$\bowtie$	$\perp$	$>$	$\ltimes$

5. [5 pts. each] Convert to base-10:

   (a) 46.3_7
   (b) $C.1D_{16}$

6. [10 pts. each] Convert to the base indicated

   (a) 23.8 to base-5 radix form.
   (b) $\frac{1945}{144}$ to base-12 radix form.

7. [10 pts. each] Perform the indicated operation.

   (a) $3.122_4 + 22.312_4$
   (b) $5.67_8 \times 5.3_8$

8. [10 pts. each] Perform the long division in the base indicated.

   (a) $403_7 \div 6_7$ (state the answer with a bar over the repeating digit)
   (b) $4233_8 \div 23_8$ (carry out to the 8^{-2} place)

9. [10 pts.] Convert $A85D_{16}$ directly to base-2.